# Seeking a Simple Complex System

G. Aeppli and P. Chandra

Not all solids display the perfect structural order shown in textbooks. Moreover, disordered systems behave in ways qualitatively distinct from their periodic counterparts (1, 2). At low temperatures, disorder can lead to a large number of minimum-energy states; as they approach equilibrium, the systems become trapped indefinitely in states that do not have the lowest energy. The ideas developed for understanding disordered systems have been applied to such diverse areas as the design of high-density computer chips (3), coding theory (4), protein-folding dynamics (5), and collective computation (6). Is disorder a prerequisite for this sort of complexity? Recent experiments on magnetic materials (7–14) suggest that it is not. The fundamental and technological implications of these new results are presently being explored in superconducting networks (15, 16) with geometries similar to those of the anomalous magnetic materials. An underlying theme has been the search for the simplest examples, which are often provided by magnetic systems.

In clean magnetic materials the magnetic ions reside on a regular lattice, and the interaction between moments is generally short-range, determined by the overlap of the electron wave functions. The key physical features can be gleaned from simplified lattice models of spins, denoted by arrows (see figure). Here only nearest neighbor interactions between spins are considered. At high temperatures, the spins at the lattice sites act independently. However, at temperatures for which the thermal and interaction energies are comparable, the spins no longer fluctuate. By analogy with the freezing of liquids to form solids, at low temperatures most periodic magnets freeze into spin crystals with well-defined long-range order. If the spin exchange favors parallel alignment of neigh-

The authors are at the NEC Research Institute, Princeton, NJ 08540, USA. E-mail: premi@research.nj.nec.com, gabe@research.nj.nec.com

boring spins, the system is a ferromagnet (see figure), like iron. By contrast, if the interaction is antiferromagnetic, it is energetically favorable for neighboring spins to be antiparallel (see figure), and there will be no net magnetization.

A spin system is frustrated if it cannot minimize the energy of each spin pair in the network simultaneously. It is strongly frustrated if it can minimize its total energy
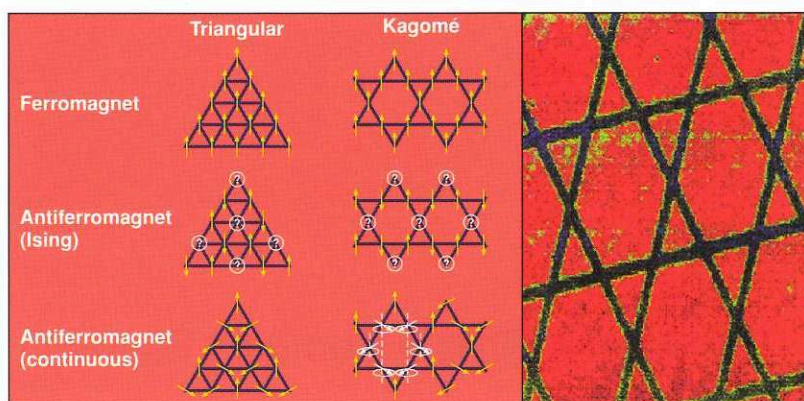
Spin control. (Left) A schematic of the triangular and Kagomé spin systems; the six white continuous spins on the Kagomé lattice can be rotated by 180° without affecting the system. **(Right)** A micrograph of a Kagomé superconducting network; each side of a hexagon is 2 μm. [Courtesy M. J. Higgins, S. Bhattacharya, P. M. Chaikin, S. Sethuraman, and R. Bojko]

equally well (or badly) in a large number of configurations. A simple example of a strongly frustrated system is an antiferromagnet on the triangular lattice (17) (see figure) where each spin can point either up or down (Ising). For this geometry it is impossible to satisfy all three spin pairs on a given triangular plaquette; thus, there is always one confused spin per triangle leading to a large number of configurations of equal energy. For the Ising model on the Kagomé lattice, a two-dimensional network of corner-sharing triangles (see figure), the number of ground states also diverges with system size (18). In principle there should be no observed spin order in such systems, and indeed the absence of spin crystallization was predicted many years ago (18). Magnetic materials avoid frustration by means of symmetry-reducing lattice distortions. Until recently the energy equality among the many states expected for candidate frustrated materials did not persist to low temperatures, and conventional spin order was always observed (19).

Interest in frustrated magnets then waned until it became clear that disordered alloys behave differently from their clean counterparts (1, 2). In dirty magnets, the spins freeze into a random configuration. Unlike conventional spin crystals, the low-temperature properties of these spin glasses are dependent on sample history, suggesting a large number of mutually inaccessible states. Qualitatively, disordered magnets have been modeled as spins on a regular lattice with random ferromagnetic and antiferromagnetic interactions, leading to frustration and complexity (1, 2).

Spin glasses resemble real glasses that solidify without crystallizing. However, spin glasses have intrinsic disorder as a result of random interactions or site dilution, or both, whereas a real structural glass emerges from a homogeneous liquid. Spin systems are usually the theorist's choice for minimal models because their basic components (spins) have no internal structure. The dream is that the detailed study of a simple theory, yet to be identified, will reveal the key features of glass formation. Most theoretical studies focus on analytically tractable, but unphysical, models with infinite-range interactions, and their relevance to real glasses remains unclear. It is thus very encouraging to have a class of clean spin systems with short-range interactions that display glassiness on practical time scales.

Renewed interest in geometrically frustrated magnets is due largely to the discovery of a magnetic compound, $SrCr_8Ga_4O_{19}$ (SCGO), whose magnetism is derived from spin-$3/2$ Cr^{3+} ions that reside primarily on Kagomé lattices (see figure). These spins are antiferromagnetically coupled with predominantly nearest neighbor interactions and can rotate continuously. In the figure, we note that such spins are ordered on the triangular lattice. Although each individual spin pair is not satisfied, the energy of each triangle is minimized by maintaining a zero sum of all its resident spins, and there are at most two possible low-temperature states for this system. By contrast, continuously rotatable spins on a Kagomé lattice (see figure) can satisfy the triangular energy constraint while making local changes to the spin configurations. Thus, the system is frustrated and can minimize its energy in a large number of distinct configurations. Even though the interspin coupling is fairly strong in SCGO, the spins do not order at temperatures well below that

associated with their interaction; this absence of spin crystallization is attributed to geometrically induced frustration (7, 8). Further studies identified a glass transition at lower temperatures, and the presence of spin freezing in the absence of long-range spin order was confirmed by neutron scattering (8). Because the initial samples did have some site disorder, the results could be ascribed to conventional random spin glass behavior. However, recent results (10) suggest that the observed glass transition remains in the limit of vanishing disorder. The presence of zero-spin triangular clusters as the building blocks of the frozen spin configuration is consistent with recent neutron data (13), thus confirming the importance of geometrically induced frustration here.

In the meantime, several other strongly frustrated materials have been identified (9, 11, 14). All have continuous spins and share the same basic building block—the triangular plaquette—for their lattices. Needless to say, each material has its strengths and weaknesses as examples of glasses (12).

What features are characteristic of glasses with and without disorder? How dependent are the properties of glasses on the range of their underlying interactions? At present any theoretical questions about short-range interactions can only be addressed numerically. Because the number of states in the systems scales exponentially with the number of spins, one is necessarily restricted to small cluster sizes. Furthermore, realistically one can only probe such a mini-glass on time scales roughly four orders of magnitude greater than its characteristic microscopic time. By contrast, experimentalists can access almost 12 more decades in time, and glasses exhibit interesting time-dependent behavior across this full range. Ideally, a detailed study of the effects of disorder versus geometry should be performed in a controlled environment where each effect can be tuned. Superconducting networks provide precisely such a setting (15, 16). Initial results on a Kagomé network agree with the absence of ordering observed in the associated materials (16). An aim here is to identify characteristic features of the many ground states for possible information storage.

Thus, there is hope that a minimal model of glassiness will be found. Just as the study of disordered spin systems has had an impact well beyond its original realm (1–3, 5, 6), we anticipate that the study of disorder-free glasses will also lead to concepts and methods of use in many areas. For example, it appears that the characterization of ground states in several nonrandom glass models is related to problems in number theory, and the associated number sequences are important for error-correcting codes (20). The identification of a minimal clean-glass model could help find physical systems for quantum computation. Again, an obscure phenomenon in magnetic materials is improving our understanding of Nature's elegant complexity, yielding insights that may result in future conceptual and technological progress.

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