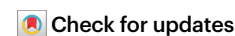


Soviet influences on Kenneth Wilson's renormalization group work

P. Chandra



Kenneth Wilson worked on the renormalization group during the Cold War, when communication between scientists in the Soviet Union and in the West was restricted. Nevertheless, Soviet physicists had a strong influence on Wilson's work.

Despite the barriers imposed by the Cold War, Kenneth Wilson's work on renormalization was significantly influenced by physics from the Soviet Union, as he acknowledged on several occasions. In his Nobel Lecture¹, Wilson cited several independent Soviet efforts^{2–10} in the pre-1971 period that were closely related to his own renormalization work. Indeed Wilson made several trips to Moscow around 1970^{11–14}, and the fact that he did so frequently (Fig. 1) suggests that he found these visits very beneficial.

Already as a young researcher, Wilson was influenced by Soviet physicists; he later admitted¹¹ to learning about renormalization in quantum electrodynamics¹⁵ from the book of Nikolay Bogoliubov and Dmitry Shirkov¹⁶, which is said to have made his thesis advisor, Murray Gell-Mann, quite annoyed. The aim of this Comment is to convey a flavor for the Soviet influences on Wilson's renormalization group work with references for the interested reader who would like to pursue more detail.

Soviet developments

It is reported that Lev Landau himself considered his mean field theory of phase transitions to be incomplete, as it could not describe ordered systems with significant fluctuations^{13,17,18}. In addition, the exact solution of the two-dimensional Ising model gives singular thermodynamic behaviour¹⁹. This cannot be reproduced using Landau theory.

In the early 1960s Alexander Voronel and his colleagues at the Institute of Physical and Technological Measurements in Moscow investigated the specific heat of argon near its critical point. They discovered an anomaly that bore striking similarity to that observed earlier at the superfluid transition of liquid helium²⁰. This removed the possibility that the critical point behaviour in helium was a quantum effect.

Voronel shared his results with many researchers including several abroad^{21–23}. This was a bold step at a time when scientific exchange between the Soviet Union and the West was restricted. Voronel had been arrested at age fourteen for his political activities²² and was known to support dissidents²⁴; he was thus most probably on a Soviet watch list. Still Voronel was eager to tell the international community about his findings. Michael Fisher recognized their importance immediately and requested Voronel's numerical data^{21–23}. Here the influence of the Cold War is evident: it seems that the Fisher–Voronel correspondence was compromised as these researchers did not receive all of each other's letters²². To prevent further loss of information, in a break with the usual



Fig. 1 | Bertrand Halperin, Kenneth Wilson and Mark Azbel in Moscow in 1977. Wilson continued to visit Moscow after the 69–70 trips discussed in this Commentary and he spoke at a 'Sunday seminar' for 'refuseniks'³² in 1977. Image courtesy of James S. Langer.

protocol of the time, Voronel included the requested numerical data at the end of a journal article so that it would be publicly accessible²². In later years Fisher often showed Voronel's specific heat singularity in his talks as a key motivation for the study of classical critical phenomena²².

At roughly the same time, Jan Sengers in the Netherlands was also challenging the conventional van der Waals–Landau approach to criticality with his transport measurements²¹. These two sets of experiments contributed to the growing collective feeling everywhere that Gaussian fluctuations around mean-field theories were not enough to describe many classical critical phenomena²¹.

Subsequent developments in the West during the 1960's have been well documented, particularly by Cyril Domb²⁵. In this Comment the focus will therefore be on the lesser-known theoretical progress in classical criticality during that time period in the Soviet physics community.

According to Alexander Polyakov^{12–14}, the modern development of this subject started with the work of Alexander Patashinski and Valery Pokrovsky^{2,3}. Their approach was inspired by a proposal by Landau in the late 1950s to express the partition function as a path integral where the Landau free energy emerges as a saddle-point solution¹⁸.

Patashinski and Pokrovsky put forward the idea that the physics of the critical point is scale-invariant^{2,3}. This produces scaling relations of the exponents describing the singularities of different thermodynamic functions at the critical point. It also explains universality, the emergence of identical singular behaviour in different systems, such as the cases of superfluid helium and argon. Similar ideas about critical phenomena were developed independently and roughly concurrently in the West by Fisher, Domb, Leo Kadanoff and their collaborators^{26–28}.

The results of Patashinski and Pokrovsky^{2,3} convinced the Soviet physics community of universality, since they had shown that the problem of classical criticality does not depend on details of a system's short-range physics^{12,14}. At the same time, work by Polyakov⁴ and by Alexander Migdal⁵ demonstrated the close connection between critical phenomena and relativistic quantum field theory.

The confluence of universality, quantum field theory and critical phenomena is apparent in a pioneering study of the four-dimensional Ising model by Anatoly Larkin and David Khmel'nitskii⁶. It was already known that Landau theory breaks down for the Ising model in four or fewer dimensions, and that the model's universal behaviour is equivalent to ϕ^4 field theory. Exploiting this link, Larkin and Khmel'nitskii applied the renormalization methods of quantum electrodynamics to the ϕ^4 model, finding clear singularities in the exponents of the specific heat and other quantities. Finally, they noted that the four-dimensional Ising model is realized in a three-dimensional uniaxial ferroelectric; here anisotropic dipolar interactions effectively add an extra dimension⁶. This was the first exact calculation of a non-mean field exponent in an experimentally realizable system, with later measurements confirming the predictions^{29,30}.

Wilson's visits to the Soviet Union

Given their shared interests in critical phenomena and relativistic field theory, it is not surprising that Wilson visited his colleagues in the Soviet Union, particularly Migdal and Polyakov¹¹, in 69–70 even though such trips were still quite unusual for US citizens at that time. Polyakov reflects that he and Migdal were very keen to learn more about Wilson's renormalization work, even though it was based on an approximate recursion scheme^{13,14}. Polyakov writes¹³: "Trying to understand it, I derived it by some crude truncation of Feynman's diagrams. Ken liked the derivation (and generously included it in his later review³¹, but I thought it just showed that the recursion formula was too primitive. However, later it helped Ken to develop a general approach to the renormalization group and epsilon expansion."

Philosophically, Wilson was not deterred by approximate expressions, particularly as he could solve them computationally; here he may well have been influenced by his father who was a theoretical chemist¹¹. Polyakov notes that quantum field theory was considered by many in the Soviet high-energy community at that time to have pathological technical issues. It was thus refreshing for both Polyakov and Migdal to see that Wilson shared their belief in the natural connection between particle physics and critical phenomena^{13,14}. Polyakov also comments¹³: "In spite of our different 'ideologies', I was very impressed by the power and depth of Ken's arguments, and learned lots of subtle things from our discussions."

During Wilson's later visits to Moscow, he also spoke at a 'Sunday seminar' for 'refuseniks'³² organized by Voronel and his colleague Mark Azbel. Refusenik was the unofficial term for a person, typically a Soviet Jew, who was denied permission to emigrate, usually to Israel. Since that time the word refusenik has entered the colloquial English lexicon to mean a person who refuses to follow the law particularly as a form of protest. In the former Soviet Union, refuseniks usually lost their jobs, which for scientists meant exclusion from their community in all forms. In Moscow, Voronel and Azbel organized regular seminars to provide mutual support and intellectual sustenance for the refusenik scientists^{24,32,33}. In 1977 Voronel and Azbel organized a meeting on collective phenomena in physics to mark the fifth anniversary of the Moscow seminar series, and Wilson was among the invited speakers from abroad. These international visitors were given a chilly welcome


by the Soviet authorities, and indeed Wilson was detained at the Moscow airport for several hours after he stated the purpose of his visit³².

Wilson clearly believed that scientific discussions with his Soviet colleagues contributed significantly to the development of his work on the renormalization group; furthermore he refused to be deterred by the tense Cold War relations between the Soviet Union and the West at the time. It seems fitting to end this Commentary with the words of Wilson himself at his Nobel Banquet³⁴: "The hardest problems of pure and applied science can only be solved by the open collaboration of the world-wide scientific community. Scientists under all forms of government must be able to participate fully in international efforts."

These sentiments continue to be important and relevant today.

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Competing interests

The author declares no competing interests.