Commodity Price Fluctuations and Monetary Policy in Small Open Economies

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Increased volatility in the world prices of commodities such as oil and food, which are basic imports for many countries, has rekindled interest on the question of how monetary policy should best adjust to external commodity price movements. Recent studies have analyzed the issue in the New Keynesian framework of Woodford (2003) and Galí (2008) adapted and extended to an open economy. As emphasized by Corsetti, Dedola, and Leduc (2010), optimal monetary policy must then balance at least two considerations. The first one is to counteract domestic distortions related to nominal price rigidities and price setting behavior. This is most critical in closed economies and, as emphasized by Woodford (2003), often results in a prescription that monetary policy should aim at the stabilization of a producer price index (PPI). The second consideration is that it can be beneficial for a small economy to use monetary policy to stabilize an international relative price such as the real exchange rate or the terms of trade. This factor, called the terms of trade externality (Corsetti and Pesenti, 2001) implies that PPI stabilization may not be optimal. Instead, it is at least theoretically possible for other monetary strategies, for example, targeting a headline inflation index such as the CPI, or even fixing the exchange rate, to dominate PPI targeting on welfare grounds.

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The question has not been settled, either in academia or in actual policy practice. In the academic arena, much of the debate has followed an influential paper by Galí and Monacelli (2005) who developed a multi-country version of the New Keynesian model and showed that, under some restrictions on parameter values, it is optimal for a small country to completely stabilize PPI inflation just as in a closed economy. This surprising result was extended by De Paoli (2009), which characterized optimal monetary policy and showed that PPI targeting was not generally optimal but remained dominant over CPI targeting and exchange rate pegging for realistic parameter values.

Both Galí and Monacelli (2005) and De Paoli (2009) abstracted from exogenous commodity price fluctuations and, hence, provide no guidance with respect to episodes of commodity price turbulence. Recent papers have filled this void. In particular, Catão and Chang (2013; 2015) have extended the Galí-Monacelli small economy framework to allow for traded commodities whose prices fluctuate exogenously. They also allow for other significant departures, such as imperfect risk sharing across countries.

In this paper, I develop a simplified version of the Catão-Chang framework with two main objectives in mind. First, I review lessons from the Catão-Chang analysis, especially conditions under which PPI stabilization coincides with or departs from an optimal (Ramsey) outcome. This review underscores that exogenous commodity price fluctuations interact with other aspects of the model including not only elasticities of demand for different goods, but also the degree of international risk sharing.

My second objective is to reexamine the question of what variables (the PPI, the CPI, the exchange rate or the output gap) should be assigned as objectives to a central bank in an open economy subject to exogenous commodity price fluctuations. My discussion is based on an exposition and critique, hopefully novel to many readers, of recent approaches to the general problem of how to compute and implement optimal monetary policy in open economies.

Following the work of Sutherland (2005), Benigno and Woodford (2006), and Benigno and Benigno (2006), the optimal policy problem is attacked by deriving a quadratic approximation to the welfare of the representative agent and expressing it in terms of deviations of endogenous variables, such as output, inflation, or the real exchange rate, from "target" values that are functions of exogenous shocks. In the model of this paper, as well as many related ones, the welfare of the representative agent can be expressed in terms of squared deviations of domestic inflation, output, and the real exchange rate from endogenously derived target values; consequently, optimal monetary policy can be expressed in terms of linear targeting rules for the real exchange rate, inflation, and output. Previous authors have noted this, particularly De Paoli (2009). I point out, however, that the welfare representation in this class of models is, in general, not unique. In fact, both the appropriate welfare criterion and the associated optimal target rules can be rewritten only in terms of inflation and output, inflation and the real exchange rate, or linear combinations of those variables (or even others); one only needs to adjust the definition of the respective targets appropriately. The practical implication is that there is no compelling reason, in terms of this analysis, to make central bank policy react to inflation, output, and the real exchange rate (as De Paoli (2009) suggests) rather than only to inflation and output, or only to inflation and the real exchange rate, as long as the policy reaction functions are designed properly.

Section 1 presents the model that serves as the framework for the analysis. A discussion of optimal monetary policy and its relation to PPI targeting is given in section 2. Section 3 discusses the second-order approximation of welfare, while a second-order approximation of the equilibrium is given in section 4. Section 5 solves for equilibrium first moments in terms of second moments. These results can be used to express the welfare function in terms of only second moments, which can then be paired with a first-order approximation of the equilibrium to find optimal policy as described in section 6. Section 7 explains how the problem can be reformulated in terms of gaps and targets from which an appropriate policy framework and optimal target rules can be applied. Section 8 discusses implications for targets and goals being assigned to the central bank, emphasizing that such reformulations are not typically unique in spite of the uniqueness of the optimal policy found in section 6. Section 9 concludes with some final remarks.

1. A Framework for Analysis

The main ideas are quite general, but it is helpful to express them in the context of a simple, concrete model. The one described in this section simplifies the one in Catão and Chang (2015), primarily in assuming one period nominal rigidities in contrast to the now popular Calvo-Yun approach, which adds realistic dynamics to the setting but obscures the essence of the optimal policy problem.

1.1 Households and Financial Markets

We study a small open economy populated by a representative household that chooses consumption and labor supply in each period to maximize $u(C) - v(N)$, where C denotes consumption, N labor effort,

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma}$$

and

$$v(N) = \varsigma \frac{N^{1+\varphi}}{1+\varphi}.$$

Even if the economy can be thought of as being infinitely lived, our assumptions here allow us to focus on a single period; therefore, we omit time subscripts.

The household takes prices and wages as given. It owns all domestic firms and, as a consequence, receives all of their profits as dividends. Finally, it may have to pay taxes or receive transfers from the government.

In order to characterize the household choice of consumption and savings, we need to describe the menu of assets available. For the most part, we follow Galí and Monacelli (2005) and most of the literature assuming that the household has unfettered access to international financial markets that, in turn, are assumed to be complete. The consequence, as it is well known, is the perfect risk sharing condition

$$C = C^* X^{1/\sigma} \tag{1}$$

where C^* is consumption in the rest of the world (ROW), assumed to be constant for simplicity, and X is the real exchange rate (the relative price of ROW consumption in terms of home consumption).[1] The intuition is that complete financial markets allow perfect sharing of risk across countries, which implies that marginal utilities of

1. To be sure, this condition is usually written as $C = \kappa C^* X^{1/\sigma}$ for some constant κ. But one can redefine world consumption as κC^* so there is no loss of generality in setting $\kappa = 1$.

consumption at home and in the ROW should be proportional up to a correction for their relative cost–the real exchange rate.

Perfect risk sharing is a drastic simplification since it ties domestic consumption to the real exchange rate. It greatly simplifies the analysis, which is the main reason to adopt it here. But most of the analysis will not hinge on that assumption. To illustrate, at the end of section 3 we sketch the consequences of the polar opposite assumption of portfolio autarky, which in this setting is equivalent to balanced trade.

The only other important choice for the household is labor effort. This is given by the equality of the marginal disutility of effort and the utility value of the real wage:

$$\frac{v^{'}(N)}{u^{'}(C)} = \varsigma N^{\varphi} C^{\sigma} = \frac{W}{P} \tag{2}$$

with W and P denoting the wage rate and the price of consumption (the CPI), both in domestic currency units.

1.2 Commodity Structure, Relative Prices, and Demand

The home consumption good is assumed to be a C.E.S. aggregate of two commodities: one of them is an imported commodity (such as food or oil) and the other is a Dixit-Stiglitz composite of differentiated varieties produced at home under monopolistic competition. This commodity structure, taken from Catão and Chang (2015), allows for the study of the role of fluctuations in world commodity prices and their interaction with nominal rigidities and monetary policy.

Cost minimization implies that the CPI is

$$P = \left[(1-\alpha)P_h^{1-\eta} + \alpha P_m^{1-\eta}\right]^{1/(1-\eta)} \tag{3}$$

where P_h is the price of home output and P_m the price of imports, both expressed in domestic currency. η is the elasticity of substitution between home goods and imports and α is a share parameter. It also follows that the demand for home produce is given by

$$C_h = (1-\alpha)\left(\frac{P_h}{P}\right)^{-\eta} C = (1-\alpha)Q^{-\eta}C \tag{4}$$

where we have defined Q as the real price of home output:

$$Q = P_h/P \tag{5}$$

Imports are available from the world market at an exogenous price P_m^* in terms of ROW currency. Assuming full exchange rate pass through, and letting S denote the nominal exchange rate, the domestic currency price of imports is then $P_m = SP_m^*$.

As in Catão and Chang (2015), the world price of imports relative to the world price of ROW consumption is random and exogenous. This captures the recent environment of fluctuating commodity prices and has a key implication for the link between the real exchange rate and the terms of trade defined as the price of imported consumption relative to the price of home produce:

$$T = \frac{P_m}{P_h}$$

The real exchange rate is defined as

$$X = \frac{SP^*}{P}$$

where P^* is the world currency price of ROW consumption. It follows that

$$T = \frac{SP_m^*}{P_h} = \frac{XZ}{Q}$$

where $Z = P_m^*/P^*$ is the world relative price of imports.

Using 3 to substitute for Q in the previous expression and rearranging, one obtains

$$XZ = \frac{T}{\left[(1-\alpha)+\alpha T^{1-\eta}\right]^{1/(1-\eta)}}$$

In the absence of fluctuations in the world relative price of imports, the preceding equation becomes a one to one correspondence between the real exchange rate and the terms of trade. This is a feature of most existing models that is often contradicted by the data. If Z is allowed to fluctuate, the correlation between the real exchange rate and the terms of trade can be less than perfect, which is not only more realistic but also has some consequences for the policy analysis.[2]

In keeping with the literature, we assume that there is a ROW demand for the home composite good which has the same form as (4):

$$\begin{aligned} C_h^* &= \alpha\left(\frac{P_h}{SP^*}\right)^{-\gamma} C^* \\ &= \alpha\left(\frac{Q}{X}\right)^{-\gamma} C^* \end{aligned}$$

where γ is the elasticity of foreign demand.

Two remarks are in order: First, I have not imposed that the elasticities of home demand and foreign demand for the home composite good be the same; almost all of the literature, however, assumes that $\eta = \gamma$.[3] Second, the foreign demand for the home composite depends on the real exchange rate and Q and, hence, the terms of trade–by (3). If monetary policy can affect these relative prices, then it can also affect the foreign demand for domestic output. This will be the source of what is called the terms of trade externality.

1.3 Production

As mentioned, a continuum of varieties of the home composite good are produced in a monopolistically competitive sector. Each variety is produced by a single firm j [0,1] via a technology

$$Y(j) = AL(j)$$

where $Y(j)$ is output of variety j, A an exogenous technology shock, and $L(j)$ labor input.

2. This is emphasized in Catão and Chang (2015).

3. On the other hand, we have chosen the constant of proportionality to be α. This is without loss of generality, as (again) one can redefine the units of world consumption if needed.

Variety producers take wages and import prices as given. For reasons discussed below, we allow for a subsidy υ to the wage in this sector, so that nominal marginal cost is

$$\Psi = \frac{(1-\upsilon)W}{A} \tag{6}$$

As mentioned, variety producers set prices in domestic currency under monopolistic competition. Catão and Chang (2015) assumed that price setting follows the well-known Calvo protocol. While that assumption imparts interesting dynamics to the model, it increases its technical complexity greatly, which obscures the basics of the policy analysis. Hence, I make the much simpler assumption here that prices are set one period in advance of the realization of exogenous shocks. The sacrifice in terms of dynamic realism will hopefully be compensated by increased insight.

With prices set one period in advance, all producers will adopt the same rule, given by

$$\mathcal{E}\left[C^{-\sigma}\frac{Y}{P}(P_h - \frac{\varepsilon}{\varepsilon-1}\Psi)\right] = 0 \tag{7}$$

where $\mathcal{E}$ is the expectation operator and Y is the level of domestic production common to all producers. The intuition is standard: under flexible pricing, each producer j would set its price as a fixed markup (of $\varepsilon/(\varepsilon - 1)$) on marginal cost; the condition above can be seen as a generalization of such a condition.

1.4 Equilibrium

Equilibrium requires that the supply of the home composite good equal the sum of home and foreign demand for it:

$$Y = C_h + C_h^* \tag{8}$$

To close the model, I assume that monetary policy determines nominal consumption expenditure:

$$M = PC \tag{9}$$

It will be useful to rewrite the equilibrium equations in a simpler way. The CPI definition (3) can be rewritten as

$$1 = (1 - \alpha)Q^{1-\eta} + \alpha\,(XZ)^{1-\eta} \tag{10}$$

Likewise, the definitions of C_h and C_h^* imply that world demand for home output can be written as

$$Y = (1 - \alpha)Q^{-\eta}C + \alpha X^{\gamma}Q^{-\gamma}C^* \tag{11}$$

Finally, the pricing rule 7 can be written as

$$P_h = \frac{\varsigma\varepsilon(1-\nu)}{\varepsilon - 1}\frac{\mathcal{E}(Y / A)^{1+\varphi}}{\mathcal{E}\left(C^{-\sigma}Y / P\right)} \tag{12}$$

Equations (9)-(12) together with (5), the perfect risk sharing condition (1), and the distribution of M, determine P, C, Y, P_h, X, and Q.

Under portfolio autarky, the balanced trade condition $P_hY = PC$, or equivalently

$$C = QY \tag{13}$$

must hold, replacing (1) in the definition of equilibrium. The other equilibrium conditions remain the same.

2. Optimal Policy, the Natural Outcome, and PPI Targeting

Intuition and a long tradition might suggest that monetary policy should aim at replicating the outcomes under flexible prices (the natural outcome). Indeed, in basic New Keynesian models of closed economies, such a prescription would achieve an optimal or Ramsey allocation. This implies that PPI targeting is an optimal policy rule since zero producer price inflation replicates the natural outcome.

In open economies, however, the Ramsey allocation coincides with the natural outcome only under very stringent circumstances. This section characterizes exactly what those circumstances are and, consequently, identifies conditions under which PPI targeting may potentially be dominated by alternative policy rules. The analysis here is very similar to that in Catão and Chang (2013), to which the reader can refer for a more detailed discussion.

2.1 The Ramsey Outcome

The economy's Ramsey problem can be defined as the maximization of the expected welfare of the representative agent subject to resource constraints and world demand. Since the choice variables can be made contingent on the realization of exogenous uncertainty, the problem is appropriately solved state by state. Hence, we can take any exogenous variables as known.

The resulting problem is to maximize $u(C) - v(N)$ subject to (10), (11) and, under perfect risk sharing, (1). To simplify, note that (10) defines the real price of home output Q as a function, say $Q(XZ)$ of XZ the real exchange rate multiplied by the world relative price of food. Keeping that in mind, and also (1), world demand can be rewritten as

$$\begin{aligned} AN &= (1-\alpha)Q^{-\eta}C + \alpha X^{\gamma}Q^{-\gamma}C^{*} \\ &= (1-\alpha)Q(XZ)^{-\eta}(C^{*}X^{1/\sigma}) + \alpha X^{\gamma}Q(XZ)^{-\gamma}C^{*} \equiv \Omega(X,Z) \end{aligned}$$

The function $\Omega(X,Z)$ expresses the total demand for home output, in general equilibrium, as a function of the real exchange rate given Z. Importantly, the elasticity of Ω with respect to the real exchange rate summarizes how demand for home output responds to a real depreciation, taking all direct and indirect effects into account. For instance, it becomes apparent that a real depreciation increases demand for home output via an increase in home consumption due to the perfect risk sharing assumption.

The objective function, in turn, can be rewritten as

$$u(C^{*}X^{1/\sigma}) - v(N)$$

under perfect risk sharing. The Ramsey problem, then, is to choose the real exchange rate X and the amount of labor effort N to maximize utility subject to $AN = \Omega(X,Z)$.

The first order condition for maximization is easy to derive and can be written as

$$\frac{1}{\sigma}Cu'(C) = \frac{X\Omega_X}{\Omega}Nv'(N) \tag{14}$$

The intuition is quite simple. A one percent real depreciation increases home consumption by $1/\sigma$ percent because of perfect risk sharing. The level of consumption, then, increases by $1/\sigma$ times C and, hence, utility increases by the LHS of the FOC. On the other side, the term $X\Omega_X/\Omega$ is the total elasticity of demand for home output with respect to X. Hence a one percent real depreciation raises the demand for home output and the level of labor effort by $X\Omega_X/\Omega$ times N. The RHS is, accordingly, the disutility of the real depreciation associated with increased demand for home goods and labor effort. For an optimal plan, the two sides must coincide.

The Ramsey outcome is then pinned down by (14) together with (1), (10), and (11). It is to be noted that these equations depend on the exogenous shocks, including Z. Hence, in general, the Ramsey outcome prescribes a time varying solution.

2.2 The Natural Outcome and Policy Implications

In the absence of nominal rigidities, producers would set prices as a markup on marginal cost:

$$P_h = \frac{\varepsilon}{\varepsilon - 1}\Psi$$

Dividing both sides by P and using (6) and (2), this reduces to

$$Q = \frac{\varepsilon(1-\nu)}{\varepsilon - 1}\frac{v'(N)}{Au'(C)}$$

or rewritten,

$$\frac{\varepsilon - 1}{\varepsilon(1-\nu)}Cu'(C) = \left[\frac{C}{QY}\right]Nv'(N) \tag{15}$$

The natural outcome is determined by this equation in conjunction with (1), (10), and (11).

It follows that the system of equations that define the Ramsey outcome differ from that underlying the natural outcome only in (14) versus (15).

This has several implications:

— For the natural outcome to be optimal it must be the case that

$$\frac{\varepsilon(1-\nu)}{\varepsilon-1}\frac{C}{QY}=\sigma\frac{X\Omega_X}{\Omega} \tag{16}$$

with Ω and Ω_X evaluated at the natural outcome. This is not the case in general, and the discrepancy will reflect the different elasticities and other aspects of the model.

— There is a discrepancy even if $\varepsilon(1-\nu)/(\varepsilon-1)=1$ that is, even if the production subsidy is adjusted to eliminate the impact of monopoly power in the steady state.

— For the special case in which $\eta=\gamma=1/\sigma=1$ the previous equation reduces to

$$\frac{\varepsilon(1-\nu)}{\varepsilon-1}=\frac{1}{1-\alpha}$$

— This implies that, in that special case, there is a value of the production subsidy under which monopolistic distortions completely offset the terms of trade externality. This is in fact the condition that Galí and Monacelli (2005) gave for PPI stabilization to be fully optimal.

— Most of the literature, focusing on monetary policy takes the subsidy ν to be a given constant. But one may instead suppose that ν can be time varying and chosen optimally. In that case, condition (16) can be taken to define the value of ν under which the natural outcome is equal to the Ramsey outcome. This observation reconciles our analysis with that of Hevia and Nicolini (2013) who argued that PPI targeting must be optimal as long as the government has access to a sufficiently rich menu of taxes and transfers.

One can now analyze how the natural allocation differs from the Ramsey outcome for different parameter values. This provides useful information, especially about how the optimality of PPI targeting depends on elasticities of demand. An extended discussion is found in Catão and Chang (2013).

Before leaving this section, two remarks are warranted. First, we might stress the sense in which the natural outcome can be associated with PPI targeting. Because of our assumptions on pricing here, the producer price P_h is predetermined and, hence, the PPI is always stabilized. However, in general, the markup is variable, being given by P_h/Ψ. Arguably, in models that incorporate Calvo-Yun pricing (and others) the most important implication of PPI targeting is not the stabilization of the price level but rather the stabilization of the markup. It is in this sense that we associate PPI targeting with flexible prices and a policy that results in a constant markup.[4]

The second remark is related to the role of international risk sharing. It is not too hard to amend the analysis in this section for the case of portfolio autarky. Since trade balance implies that $C = QY = QAN$, for example, the world demand function can be written as

$$AN = (1-\alpha)Q^{1-\eta}AN + \alpha X^{\gamma}Q^{-\gamma}C^{*}$$

which, since $Q = Q(XZ)$ clearly defines $Y = AN$ as an implicit function of X and Z. The first-order condition for the Ramsey plan is given by (14), except that the term $X\Omega_X/\Omega$ refers to the elasticity of the function just defined with respect to X. The analysis becomes more complex but the analysis of the determinants of policy can be amended accordingly in an intuitive way. Again, see Catão and Chang (2013) for a full development.

3. Approximating Welfare

To obtain further lessons, one may follow the literature in studying a second-order approximation to welfare. Such an approximation is obtained as follows: one can show that, in second order,

$$u(C) = u(\bar{C}) + \bar{C}u'(\bar{C})[c + \frac{1-\sigma}{2}c^2] + \mathcal{O}^3$$

where $\bar{C}$ is the non-stochastic steady-state value of consumption and $c = \log C - \log \bar{C}$ is the log deviation of consumption from its non-stochastic steady state. Also, $\mathcal{O}^3$ refers to terms that are at least cubic in C and, hence, negligible in a second-order approximation. Such terms will be omitted in the rest of the paper, although, the reader should keep them in mind at certain points.

4. In fact, a policy that ensures that equation (15) holds ex post must result in the flexible price outcome. But such a policy would then stabilize the markup P_h/Ψ.

Likewise, with a similar notation,

$$v(N) = v(\bar{N}) + \bar{N}v^{'}(\bar{N})\left[n + \frac{1+\varphi}{2}n^2\right]$$

Hence, aside from an irrelevant constant, $u(\mathrm{C}) - v(N)$ is proportional to

$$c + \frac{1-\sigma}{2}c^2 - \frac{\bar{N}v^{'}(\bar{N})}{\bar{C}u^{'}(\bar{C})}\left(n + \frac{1+\varphi}{2}n^2\right)$$

In steady state, one can show that

$$\frac{\bar{N}v^{'}(\bar{N})}{\bar{C}u^{'}(\bar{C})} = \frac{\bar{N}}{\bar{C}}\frac{\bar{W}}{\bar{P}} = \frac{\varepsilon - 1}{\varepsilon(1-\nu)}$$
$$\equiv \mu$$

so that the term is a measure of the steady-state distortion associated with monopolistic competition. For notational convenience, we will denote the term by μ. The literature has focused on two cases: when the subsidy v is adjusted to compensate for domestic monopoly power in steady state $\mu = 1$ or the Galí-Monacelli case $\mu = 1 - \alpha$. Regardless, the welfare objective can be then written as

$$\mathcal{W} = \mathcal{E}\left\{c - \mu n + \frac{1}{2}[(1-\sigma)c^2 - \mu(1+\varphi)n^2]\right\} \tag{17}$$

Naturally, social welfare increases with expected consumption and falls with expected labor effort. It also falls with the variance of consumption and labor supply.[5]

The presence of the expected values $\mathcal{E}c$ and $\mathcal{E}n$ is inconvenient because, as Woodford (2003) has stressed, it means that one cannot simply use a first order, log linear approximation to the model's equilibrium in order to evaluate the welfare objective correctly in

5. Notice that $EC = E(\bar{C} + (C - \bar{C})) = \bar{C}E(1 + c + c^2/2)$ in second order. Hence, the term $E(c + c^2/2)$ captures that utility increases with expected consumption. The impact of consumption variability is $-1/2\sigma Ec^2$ and hence always negative.

second order.[6] Notice that, if ν is assumed to correct for monopoly power ($\mu = 1$), this issue disappears in a closed economy since, then, the term $c - n = y - (y - a) = a$ which is independent of welfare and, hence, can be dropped. In an open economy, in contrast, c and y do not generally coincide, and one cannot apply the same argument.

One solution to this issue developed by Sutherland (2005), Benigno and Woodford (2006), Benigno and Benigno (2006), and others, is to express $\mathcal{E}c$ and $\mathcal{E}n$ as functions of only quadratic terms from a second-order approximation of the equilibrium equations. Then one can rewrite the objective as a function of only quadratic terms. We develop this procedure next.

4. A Second-Order Approximation of the Equilibrium

I assume hereon that A and C^* are constant and equal to one, so the only uncertainty concerns the realizations of Z and M. It will be seen that the arguments are straightforward to generalize for the case in which A and C^* are also random.

As mentioned, the equilibrium equations are given by (1), (5), and (9)-(12). Of those, (1), (5), and (9) are linear in logs and, therefore, require no approximation:

$$\sigma c - x = 0, \tag{18}$$

$$q - p_h + p = 0, \tag{19}$$

$$p + c = m. \tag{20}$$

The CPI definition (10) is not log linear, so it must be approximated. One can show that, in second order,

$$(1 - \alpha)q + \alpha x = -\alpha z + \lambda_x \tag{21}$$

6. This is so because a linear approximation to the model would be correct up to a second-order residual. So inserting, say, the resulting expression for c in the welfare objective would insert a second-order residual in the objective, which cannot be ignored (since a quadratic approximation to welfare is intended to be correct up to a residual of third or higher orders).

where I have gathered second-order terms in

$$\lambda_x = -\frac{1}{2}\left[(1-\alpha)(1-\eta)q^2 + \alpha(1-\eta)(x^2+z^2)\right] - \alpha(1-\eta)xz.$$

Some remarks are warranted here. As mentioned, the presence of the commodity price shock z introduces a time varying wedge between the real exchange rate x and other international relative prices such as q. Equation (21) says that the relation between x and q is also affected by their variances, the variance of z, and the covariance between x and z. This would be ignored in a first order approximation, which would treat λ_x just as if it were zero.

The world demand for domestic output (11) can be approximated in second order by

$$y + \theta q - (1-\alpha)c - \gamma\alpha x = \lambda_y \tag{22}$$

where I have collected second-order terms in

$$\lambda_y = -\frac{1}{2}y^2 + \frac{1}{2}\left[(1-\alpha)\eta^2 + \alpha\gamma^2\right]q^2 + \frac{1}{2}(1-\alpha)c^2 + \frac{1}{2}\gamma^2\alpha x^2 - \eta(1-\alpha)qc - \alpha\gamma^2 qx$$

and defined

$$\theta = (1-\alpha)\eta + \alpha\gamma$$

The parameter θ can be regarded as the elasticity of the total demand for home output with respect to its domestic real price q given the exchange rate. A one percent increase in q reduces home demand for home produce by η percent. In addition, given x, a one percent increase in q is also a one percent increase in the world price of home output, which results in a fall in world demand by γ percent.

Finally, the second-order approximation to the pricing condition (12) is

$$p_h = \mathcal{E}\left[\varphi y + \sigma c + p + \lambda_p\right] \tag{23}$$

with

$$\lambda_p = \frac{1}{2}\left\{(1+\varphi)^2 y^2 - (y - \sigma c - p)^2\right\}.$$

The preceding expression says that p_h increases with expected demand. This is intuitive, as demand determines output and hence, labor effort, wages, and marginal costs. Hence, if expected demand goes up, firms increase prices to maintain the desired markup over marginal costs. Likewise, if expected consumption goes up, the expected wage and marginal costs go up because the marginal utility of the wage falls, resulting in a fall in expected labor supply.

Less obviously, the term λ_p reflects that firms choose nominal prices as a hedge against uncertainty. If, for instance, demand or consumption become more variable (as reflected in an increase in $\mathcal{E}y^2$), the volatility of marginal costs increase for the reasons just mentioned, inducing firms to reduce expected output by increasing prices.

5. Solving for Expected Values

As stressed by Sutherland (2005), it is now straightforward to solve for expected values of all variables as functions of second moments. Let $V = (y\ c\ q\ p\ p_h x)'$ denote the column vector of endogenous variables, and $\Lambda = (0\ 0\ 0\ \lambda_x\ \lambda_y\ \lambda_p)'$ collect second moments. With some loss of generality, we assume that $\mathcal{E}z = \mathcal{E}m = 0$.[7]

Then, taking expectations in the second-order system derived in the previous section, one can collect the six equations (18)-(23) in an expression such as $\Gamma\mathcal{E}V = \mathcal{E}\Lambda$ with the matrix Γ given by the coefficients of the left hand sides of the second-order approximation equations. Expected values are then given by $\mathcal{E}\mathrm{V} = \Gamma^{-1}\mathcal{E}\Lambda$. Therefore, in general, it is relatively straightforward to express first moments as functions of second moments. One can use that result in order to remove $\mathcal{E}c$ and $\mathcal{E}n = \mathcal{E}y$ from the objective function $\mathcal{W}$, thus arriving at the desired purely quadratic objective.

In our case, the simplicity of the model allows us to solve the necessary system by hand. Taking expectations, the perfect risk

7. The first equality is a normalization. The second one entails no loss of generality since money is neutral in this model.

sharing condition becomes $\sigma\mathcal{E}c = \mathcal{E}x$ while $\mathcal{E}q = p_h - \mathcal{E}p$. Inserting these two expressions into the pricing equation yields

$$\mathcal{E}q = \varphi\mathcal{E}y + \mathcal{E}x + \mathcal{E}\lambda_p.$$

Taking expectations in the CPI definition gives

$$(1-\alpha)\mathcal{E}q + \alpha\mathcal{E}x = \mathcal{E}\lambda_x,$$

and the demand for home output in expectations becomes

$$\mathcal{E}y + \theta\mathcal{E}q - \Psi\mathcal{E}x = \mathcal{E}\lambda_y$$

where

$$\Psi = \frac{(1-\alpha)}{\sigma} + \gamma\alpha.$$

The parameter Ψ can be seen as the elasticity of demand for home output with respect to the real exchange rate, other prices given. It reflects that a one percent increase in x leads to a $1/\sigma$ increase in home consumption because of perfect risk sharing, and a one percent fall in the world price of home output, leading to an increase in world demand by γ percent.

These three equations can be solved readily for $\mathcal{E}y, \mathcal{E}q, \mathcal{E}x,$ and $\mathcal{E}c$ as functions of the expected λ_x, λ_y and λ_p. The solution has the form

$$\mathcal{E}y = \mathcal{E}\left[\phi_{yy}\lambda_y + \phi_{yp}\lambda_p + \phi_{yx}\lambda_x\right]$$

$$\mathcal{E}c = \frac{1}{\sigma}\mathcal{E}x = \mathcal{E}\left[\phi_{cy}\lambda_y + \phi_{cp}\lambda_p + \phi_{cx}\lambda_x\right]$$

where

$$\phi_{yy} = 1/(1+\varphi\Theta),\ \phi_{yp} = -\Theta\phi_{yy},\ \phi_{yx} = -(\theta - \Psi)\phi_{yy},$$

$$\phi_{cx} = \phi_{yy}(1+\varphi\theta)/\sigma,\ \phi_{cp} = -\phi_{yy}(1-\alpha)/\sigma,\ \phi_{cy} = -\phi_{yy}(1-\alpha)/\varphi\sigma,$$

and we have defined

$$\Theta = \alpha\theta + (1-\alpha)\Psi$$
$$= \Psi + \alpha(\theta - \Psi) = \Psi + \alpha(\eta - \frac{1}{\sigma})$$

The preceding expressions take explicit accounting of uncertainty and show how $\mathcal{E}y$ *and* $\mathcal{E}c$ are related to second moments and uncertainty. Of course, these are not yet solutions to expected values, since λ_x, λ_y and λ_p are functions of endogenous variables. Note that these expressions would be set to zero in a first order approximation.

Expected welfare can then be written as

$$\mathcal{W} = \mathcal{E}\begin{bmatrix} (\phi_{cy} - \mu\phi_{yy})\lambda_y + (\phi_{cp} - \mu\phi_{yp})\lambda_p \\ +(\phi_{cx} - \mu\phi_{yx})\lambda_x + \frac{1}{2}[(1-\sigma)c^2 - \mu(1+\varphi)n^2] \end{bmatrix}, \tag{24}$$

which is purely quadratic, as we had sought.

To illustrate, take the case $\eta = 1/\sigma$ with $\mu = 1$ which has been emphasized in the literature. In that case, $\theta = \Psi = \Theta$ and the expected linear terms in the objective function simplify considerably and become

$$\mathcal{E}c - \mathcal{E}n = \mathcal{E}c - \mathcal{E}y = \frac{\alpha\gamma}{1+\theta\varphi}\mathcal{E}\lambda_p - \frac{\eta(1-\alpha)\varphi + 1}{1+\theta\varphi}\mathcal{E}\lambda_y + \eta\mathcal{E}\lambda_x$$

6. A Linear-Quadratic Approximation to Optimal Policy

As mentioned, a great advantage of having expressed the objective, $\mathcal{W}$, as a purely quadratic term is that it allows the remainder of the analysis to be carried out by looking only at the linear approximation of the model. This is because the residuals associated with the linear approximation, which can be of order two, become terms of third order and higher when taking squares and cross products, so they can be ignored legitimately in second order.

The first order system is obtained from the second-order equations by simply setting λ_x, λ_y and λ_p equal to zero, and becomes

$$\sigma c - x = 0 \tag{25}$$

$$q - p_h + p = 0 \tag{26}$$

$$p + c = m \tag{27}$$

$$(1 - \alpha)q + \alpha x = -\alpha z \tag{28}$$

$$y + \theta q - (1 - \alpha)c - \gamma\alpha x = 0 \tag{29}$$

$$p_h = \mathcal{E}[\varphi y + \sigma c + p] \tag{30}$$

To proceed, take expectations of all equations. It then follows that all variables have an expectation of zero. Hence, $p_h = 0$ in first order, and we can, in practice, forget about (30).

Also, m appears only in (27). This means that we can think of the price level p as the policy variable, letting (27) tell us the associated value of m. This allows us to forget about m altogether in the spirit of the "cashless economy" analysis popularized by Woodford (2003).

Since $p_h = 0$ (26) says that $q = -p$ in first order, therefore, we can equivalently take q as the policy variable.

For concreteness, take p as the control variable. Using $q = -p$ to eliminate q from the system; (28) then gives

$$x = \left(\frac{1}{\alpha} - 1\right)p - z.$$

Consumption is then

$$c = \frac{1}{\sigma}x = \frac{1}{\sigma}\left[\left(\frac{1}{\alpha} - 1\right)p - z\right]$$

$$y = \frac{\Theta}{\alpha}p - \Psi z \tag{31}$$

These expressions can now be used to express λ_x, λ_y and λ_p, as well as c^2 and y^2, in terms of the squares and cross products of p and z.

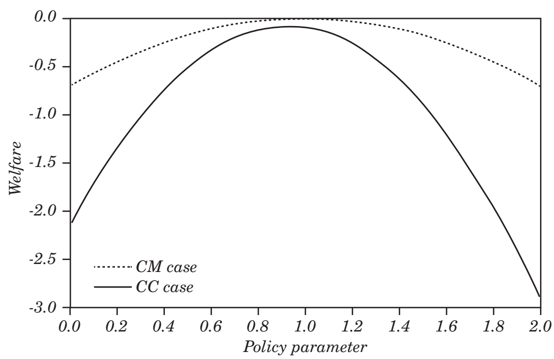
The optimal policy problem can then be seen as one of choosing the distribution of p to minimize the resulting expression for $\mathcal{W}$ as given by (24). Since the problem is now linear-quadratic, the solution will be linear in the shock z : $p = \bar{\kappa} z$, for some constant $\bar{\kappa}$ which is now straightforward to find.

The optimal solution could be compared with any given policy. For example, one could define CPI targeting as a policy that sets $p = 0$. Likewise, a policy that stabilizes the real exchange rate would set $x = 0$; note that this also results in a linear rule in the form $p = \kappa z$ with $\kappa = -\alpha/(1-\alpha)$. A third option would stabilize domestic markups, which is the hallmark of PPI targeting. From (30), such a policy must set $\varphi y + \sigma c + p = \varphi y + x + p = 0$. With the expressions above, this would require $p = \kappa_{PPI} z$ with

$$\kappa_{PPI} = \frac{\alpha[1+\varphi\Psi]}{1+\varphi\Theta} \tag{32}$$

In general, the policies just mentioned will differ from each other and from the optimal policy. One could now explore how the discrepancies depend on the values of different parameters such as elasticities of demand or the coefficient of relative risk aversion σ.

Figure 1. Policy Rules and Welfare



Source: Author's calculations.

To illustrate, figure 1 displays the welfare consequences for alternative policies of the form require $p = \iota\, \kappa_{PPI} z$ where ι varies between zero and two along the horizontal axis. Hence, $\iota = 1$

corresponds to PPI targeting, and $\iota = 0$ to CPI targeting. For each such policy, welfare can be computed to be proportional to the standard deviation of z the proportionality constant is plotted along the vertical axis.

Two parameterizations are examined. Both assume $\alpha = 0.3$ and $\varphi = 1$, which are standard values. Also, μ is set at $1 - \alpha$ so that PPI targeting is theoretically optimal in the unit elasticity case of Galí and Monacelli (2005). So figure 1 plots welfare for that case, $\sigma = \eta = \gamma = 1$, labeled "GM case." As an alternative, the figure also displays a plot for the "CC case" of $\sigma = 3$, $\eta = 0.2$, $\gamma = 5$. This combination of parameters is prominently featured in Catão and Chang (2015), who argue that it is a reasonable approximation of an economy that imports food or oil (so that the elasticity of substitution between domestic goods and imports is low) but exports goods with a relatively elastic world demand.

For the GM case, the figure gives the expected results. Welfare is maximized at $\iota = 1$, confirming that PPI targeting is optimal. But that is a special case. For the CC parameterization, the figure shows that PPI targeting is dominated by other policies. Welfare is maximized at $\iota = 0.92$; this result, in particular, indicates that, as we move away from the GM case, optimal policy places more emphasis on terms of trade externality rather than domestic monopoly distortions.

This suggests that one should explore, in more detail, how optimal policy departs from PPI targeting under more realistic assumptions (e.g. a multi-period setting), and allow for other parameterizations. This topic is developed in Catão and Chang (2013; 2015). Catão and Chang's model is essentially the same as discussed here, except that they assume Calvo pricing, which imparts non trivial dynamics and allows for productivity and monetary policy shocks in addition to commodity price shocks. One of their main findings is that the optimality of PPI targeting, vis a vis other rules (especially the targeting of forecasted CPI), depends crucially on various parameters such as the elasticity of foreign demand for home output (γ) and the structure of financial markets. With complete international risk sharing, PPI targeting delivers lower welfare than expected CPI targeting, except for unrealistically low values of γ. However, PPI is a superior choice under portfolio autarky and, more generally, under even mild degrees of imperfections in international risk sharing. For a complete discussion and details, the reader is referred to Catão and Chang (2013; 2015).

7. Policy Targets, Gaps, and Optimal Rules

Benigno and Benigno (2006), De Paoli (2009), and others have proposed an alternative perspective on the optimal policy problem based on rewriting the social objective function $\mathcal{W}$ in terms of "targets" and "welfare-relevant gaps." An associated implication is that optimal policy can be expressed as a "flexible targeting rule." One of the advantages of such an approach, these authors have argued, is that it identifies targets that should be assigned to a central banker in order to maximize social welfare. It also has the virtue of reconciling recent theory with a venerable tradition of loss functions that are quadratic in inflation and deviations of inflation and perhaps other variables from targets.

In our context, it will be useful to separate a role for ex-post inflation (nominal) variability from the role of real variables. To do this, observe that the price level p does not appear in the Λ terms except for λ_p, which can be rewritten as

$$\begin{aligned}\lambda_p &= \frac{1}{2}\left\{\varphi(2+\varphi)y^2 - (\sigma^2c^2 + p^2 - 2y\sigma c - 2yp + 2\sigma pc)\right\}\\ &= \frac{1}{2}\left\{\varphi(2+\varphi)y^2 - (\sigma^2c^2 - 2y\sigma c + 2qy - 2\sigma qc)\right\} - \frac{1}{2}p^2\\ &\equiv \tilde{\lambda}_p - \frac{1}{2}p^2\end{aligned}$$

the next to last replacement using the fact that $q = -p$ in first order.

This implies that the objective function can be rewritten, using (24) as

$$\mathcal{W} = \mathcal{E}\begin{bmatrix}(\phi_{cy} - \mu\phi_{yy})\lambda_y + (\phi_{cp} - \mu\phi_{yp})\tilde{\lambda}_p\\ +(\phi_{cx} - \mu\phi_{yx})\lambda_x + \frac{1}{2}[(1-\sigma)c^2\\ -\mu(1+\varphi)n^2]\end{bmatrix} - \frac{1}{2}(\phi_{cp} - \mu\phi_{yp})\mathcal{E}p^2$$

plus a term in z^2 and, therefore, independent of welfare. Noting that λ_y, λ_x and λ_p depend only on the vector of real variables $\tilde{V} = (y, c, q, x)'$, the preceding can be written as

$$\mathcal{W} = -\mathcal{E}\left[\left(\frac{1}{2}\tilde{V}'D\tilde{V} + \tilde{V}'Fz\right) + \frac{1}{2}w_p p^2\right]$$

for appropriately chosen matrices D and F and $w_p = (\phi_{cp}\text{-}\mu\phi_{yp})$.

The preceding representation is suggestive as it rewrites the welfare objective as an expected loss function which depends on inflation variability with weight w_p and a component that depends only on the volatility of real variables. Hence, it emphasizes that stable inflation should be an objective of monetary policy but, generally, should not be the only one: the real variables included in the vector $\tilde{V}$ also matter for welfare.

A further simplification is available from the observation that the three first order equations (25), (28), and (29) can, in principle, be solved for any three of the real variables included in the vector $\tilde{V}$ in terms of the fourth one and the shock z. For instance, adding the identity $y = y$ as a fourth equation $\tilde{V}$ one can write

$$\Phi_y \tilde{V} = \psi_y y + \psi_z z, \tag{33}$$

where

$$\Phi_y = \begin{bmatrix} 0 & \sigma & 0 & -1 \\ 0 & 0 & 1-\alpha & \alpha \\ 1 & -(1-\alpha) & \theta & -\gamma\alpha \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\psi_y = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}'$$

and

$$\psi_z = \begin{bmatrix} 0 & -\alpha & 0 & 0 \end{bmatrix}'$$

The matrix Φ_y is invertible and, hence, one can write

$$\tilde{V} = N_y y + N_z z$$

where $N_y = \Phi_y^{-1}\psi_y$ and $N_z = \Phi_y^{-1}\psi_z$. Therefore, as mentioned, the vector $\tilde{V}$ can be expressed as a function of only y and z. Note, at this point, that one could have equally expressed $\tilde{V}$ in terms of z and a different variable, say, the real exchange rate x.

Inserting the last equation into the "real" part of $\mathcal{W}$:

$$\begin{aligned}\frac{1}{2}\tilde{V}'D\tilde{V}+\tilde{V}'Fz &= \frac{1}{2}(N_y y+N_z z)'D\left(N_y y+N_z z\right)+\left(N_y y+N_z z\right)'Fz \\ &= \frac{1}{2}(N_y' DN_y)y^2+(N_y' DN_z+N_y' F)yz+t.i.p. \\ &= \frac{1}{2}w_y[y^2-2\varpi yz]+t.i.p.\end{aligned}$$

where w_y and ϖ are scalars defined in the obvious way and, following Woodford (2003), *t.i.p.* stands for terms independent of policy, which are irrelevant for the definition of the welfare objective.

This suggests one further rewriting:

$$y^2-2\varpi yz=(y-y^T)^2+t.i.p.$$

where

$$y^T=\varpi z$$

is the linear function of the shock determined by the parameter ϖ. Replacing it in the objective function, we finally obtain (dropping *t.i.p* terms)

$$\mathcal{W}=\frac{1}{2}\mathcal{E}\left[w_y(y-y^T)^2+w_p p^2\right] \tag{34}$$

This expression for $\mathcal{W}$ emphasizes that monetary policy should seek to minimize a weighted sum of deviations of inflation from a zero mean and deviations of output from $y^T=\varpi z$ (sometimes called the welfare relevant output gap). The random variable y^T is then appropriately seen as a target of policy; in a sense, this provides a justification for flexible inflation targeting.

The main constraint in the maximization is (31), which can be seen as the Phillips Curve in this model and can be rewritten as

$$y-y^T=\frac{\Theta}{\alpha}p-(\Psi+\varpi)z \tag{35}$$

Maximizing (34), subject to this constraint, now gives

$$(y - y^T) + \varkappa p = 0$$

with $\varkappa = \alpha w_p / \Theta w_y$. This can be seen as a flexible targeting rule. It emphasizes that the central bank's optimal policy reflects a trade-off between deviations from a zero inflation target and a nonzero welfare relevant output gap.

The form of the objective function $\mathcal{W}$ and the targeting rule are the same as the ones that Woodford (2003), Galí (2008), and others have proposed as optimal for a closed economy. Hence, our discussion suggests that central banks in small open economies should be given the same objectives and follow the same rules as their counterparts in closed economies.

Such an interpretation, however, would be too simplistic and perhaps misleading for a number of reasons. First, the welfare function just derived is a transformation of the utility function of the representative agent where we have used (second-order) approximations to the equilibrium to replace the original arguments of that function (consumption and labor effort). Naturally, the weights w_y and w_p will depend, in general, on the basic parameters of the economy, including the degree of openness, trade elasticities, technology parameters, preference parameters, and the degree of international risk sharing.

Likewise, the target $y^T = \varpi z$ is a function of the exogenous shocks to world commodity prices. In addition, the parameter ϖ is a function of other basic parameters of the economy as the derivation makes clear. In the closed economy, it is often the case that the appropriate target for output is given by natural output. But this is not the case here. In general, the target y^T found here will be different from the flexible price value of output implied by (32).

To illustrate, table 1 gives the values of w_y, w_p, ϖ, and $\varkappa$ for different parameter values. In the table, parameter values change from the unit elasticity case of Galí and Monacelli given in the first row to the Catão-Chang case in the last row in order to trace the influence of specific parameters.

The Galí-Monacelli case is interesting since the derived coefficients seem counterintuitive at first. Since $w_p = 0$, the implied central bank loss function gives no weight to inflation. Likewise, $\varkappa = 0$ so that inflation does not appear in the optimal "flexible targeting rule," which reduces to $y - y^T = 0$, that is, to stabilize

Table 1. Optimal Policy Objectives, Targets and Rules

	w_y	w_p	ϖ	$\varkappa$
$\sigma = \eta = \gamma = 1$ (GM case)	1.40	0.00	0.00	0.00
$\sigma = 3,\ \eta = \gamma = 1$	1.05	0.16	0.06	0.08
$\sigma = 3,\ \eta = 1,\ \gamma = 5$	0.37	0.38	0.17	0.17
$\sigma = 3,\ \eta = 0.2,\ \gamma = 5$ (CC case)	0.65	0.35	-0.00	0.10

Source: Author's calculations.

output around its target level. In turn, $y^T = 0$. All of this may be puzzling since we know that PPI targeting is optimal in this case. The mystery goes away when one realizes that keeping $y = y^T = 0$ requires $p = [\alpha(\Psi + \varpi)/\Theta]z$ in order to satisfy (35), which is the fixed markup condition associated with PPI stabilization.

The second row is the GM case, except that $\sigma = 3$. An implication is that w_y falls and w_p increases in the loss function. Correspondingly, $\varkappa$ becomes positive so that the flexible targeting rule pays more attention to CPI stabilization relative to output stabilization. This echoes results in Catão and Chang (2015) and reflects that a larger σ increases the importance of the terms of trade externality so that optimal policy tilts towards stabilizing the real exchange rate. Notably, also, the output target becomes $y^T = 0.06z$, expressing that it is socially beneficial for home output and labor effort to expand in response to an increase in the world price of imports. A main point here is that all of the parameters of the linear quadratic social planning problem and, hence, the solution, depend on the economy's more basic elasticities.

The third row assumes $\gamma = 5$. In that case, the linear quadratic loss function places almost the same emphasis on inflation as on the output gap. The output target is now $y^T = 0.17z$, suggesting that the output should react even more strongly than in the previous case to a shock in imports prices. On the other hand, the flexible targeting rule parameter rises to 0.17 indicating that, in the end, CPI stabilization receives more weight. Since $q = -p$ this also means that real exchange rate stabilization receives more emphasis. Overall, the intuition is that a more elastic foreign demand for home output leads to even more emphasis on the terms of trade externality relative to PPI stabilization.

Finally, the last row shows the CC case, which assumes $\eta = 0.2$. A smaller value of η means that imports and domestic goods are less substitutable in domestic consumption. The result is that the welfare function and the flexible targeting rule pay somewhat less attention to the terms of trade externality, and more attention to addressing domestic price distortions.

8. What Variables Should the Central Bank Target?

In small open economies, especially those subject to fluctuations in the world prices of food, oil, and other commodities, a debate often emerges regarding the variables that the central bank should try to stabilize. Some debate participants have sought guidance from the recent academic literature, interpreting it as implying that an optimal flexible inflation targeting should include only inflation, and the output gap.

For example, Svensson (2008) argued the following:

> "But what price index should inflation targeting ideally refer to? Recent work by Kosuke Aoki, Pierpaolo Benigno and others have emphasized that (from a welfare point of view) monetary policy should stabilize sticky prices rather than flexible prices. These results can be interpreted as favoring a core CPI or domestic inflation targeting. How should the central bank respond to oil price changes (or any terms of trade changes)? Good monetary policy is flexible inflation targeting that can be narrowly specified as aiming at both stabilizing inflation around an inflation target and stabilizing the output gap around zero. Importantly, under inflation targeting, the exchange rate is not a target variable and there is no target exchange rate level."

Our analysis in the previous section might be construed as confirming the views of Svensson (2008) and others, but in fact, perhaps surprisingly, it does not adhere to those views completely.

The key observation is that the previous section's representation of $\mathcal{W}$ as a function of an output gap and inflation can be replaced by one with $\mathcal{W}$ being written as a function of, say, a "real exchange rate gap" and inflation, or a consumption gap and inflation, and so on. This is easily seen by retracing the steps leading to (34). Specifically, we noted that the three equations (25), (28), and (29) allowed us to express any three of the four real variables (c, y, q, x)

in terms of the fourth one. In writing (33), we proceeded to express (c, q, x) in terms of y. But we could have equally expressed (c, q, y) in terms of x : adding the identity $x = x$ to (25), (28), and (29), we could have written

$$\Phi_x \tilde{V} = \psi_x x + \psi_z z$$

with $\psi_x = \psi_y$ and Φ_x equal to Φ_y except for its last row–which would be given by (0, 0, 0, 1).

It is now obvious that such a choice would lead to an objective function of the form $\mathcal{W} = -1/2\ \mathcal{E}[w_x(x - x^T)^2 + w_p p^2]$ and a target rule of the form $(x - x^T) + \varkappa p = 0$ where $x^T = \varpi z$ however, the parameters ϖ, w_x, w_p,, and $\varkappa$ would be different in this case.

In other words, there are several equivalent ways to represent the social welfare function and associated constraints and, correspondingly, many seemingly different but equivalent ways to implement an optimal monetary policy. As a consequence, one can assign the central banker an output objective, or an exchange rate objective, or a domestic producer price objective, or all of the above, as long as the meaning of "objective" is defined properly in terms of the underlying shocks that affect the economy. Note that the argument is quite general (in the end, all of the equivalent representations of the social welfare function are ultimately derived from $u(C) - v(N)$ or its second-order approximation (17).

While these observations fall quite easily from our analysis and, indeed, some readers may find them trivial, they are quite important from a practical perspective. As illustrated by the Svensson quote above, it is frequently argued that the central bank should "react to domestic inflation rather than headline inflation," or that monetary policy should "depend on the real exchange rate in addition to inflation and the output gap," or even that the central bank should have "competitiveness and the real exchange rate as one of their objectives." On the basis of the analysis here, which is representative of the recent literature, one must conclude that each and every one of these claims is right and wrong (or, at best, incomplete) at the same time. The analysis establishes that it can be optimal for the central bank to be assigned an output target and zero inflation as objectives, and to follow a rule targeting inflation and an output gap. But such a prescription is incomplete unless it specifies how the output target is defined in terms of the exogenous shocks hitting the economy, and how to compute the relative weights in the central bank's loss function and the target rule. It can be equally optimal to assign an exchange rate target to the central bank instead of, or even in addition to, an output target, as long as the target (or targets) and weights are redefined appropriately as described in this section.

We then conclude that, by itself, our analysis does not provide a rationale for telling central bankers to stabilize the output gap and PPI inflation rather than CPI inflation or the real exchange rate. Any of these variables or others can, in principle, be a suitable target of policy.

One might further ask if there are some other considerations outside the kind of analysis reviewed here that could justify why a central bank should target some variables instead of others. This question is beyond the scope of the present paper, but one may speculate that answers may be based on how the different variables relate to the credibility of the central bank.

More specifically, it may be better to target some variables rather than others from a "transparency" perspective. Arguably, a (nominal) exchange rate target is more transparent than a "domestic inflation" target just because the exchange rate is more easily and more readily observable than a domestic inflation index, especially in economies that are heavily exposed to international relative price fluctuations.

Likewise, an argument in favor of targeting the exchange rate rather than inflation could be based on the different strengths of these variables as commitment devices in the presence of time inconsistency. The announcement of an inflation target is, arguably, not as "hard" as announcing an exchange rate target, partly because sometimes the meaning of "inflation" has not been precisely defined (witnesses debate about whether inflation targets actually referred to core, versus headline, inflation). In contrast, an exchange rate target is often unambiguous.

A third line of argument may be that some variables are observed more frequently and with a shorter lag than others. Output and inflation measures, including the exchange rate and interest rates, are not as readily available as asset prices.

Notably, these arguments tend to favor exchange rates as targets rather than inflation and the output gap. Opposing arguments can indeed be constructed. For instance, one might point out that exchange rates and other asset prices are often affected by bubbles and self-fulfilling expectations and are, therefore, unreliable indicators of the economy's fundamentals. Clearly, much more research remains to be done on these issues.

9. Final Remarks

Our discussion has abstracted from the question of how central banks might implement policy in practice. The current approach to this issue is to derive and characterize the implications of policy rules such as Taylor rules. I prefer not to expand on this question here, partly because there is no satisfactory definition of an interest rate in this paper's model. Also, this issue is explored in detail in Catão and Chang (2015).

This being said, the analysis of policy rules may provide another way to discriminate among the variables that a central bank may target. Theoretical analyses frequently assume rules that make the interest rate react to measures of inflation and the output gap. Both measures are, however, nontrivial to construct and usually obtained after some lags. In contrast, asset prices, including exchange rates, are observed much more easily and quickly. Hence, if one can find a rule that makes the interest rate react to exchange rates and delivers the same allocation as a rule based on the output gap, then one should presumably prefer the former. But this obviously warrants more research.

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