Optimal Foreign Reserves and Central Bank Policy Under Financial Stress

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Rutgers and NBER

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Introduction and Motivation

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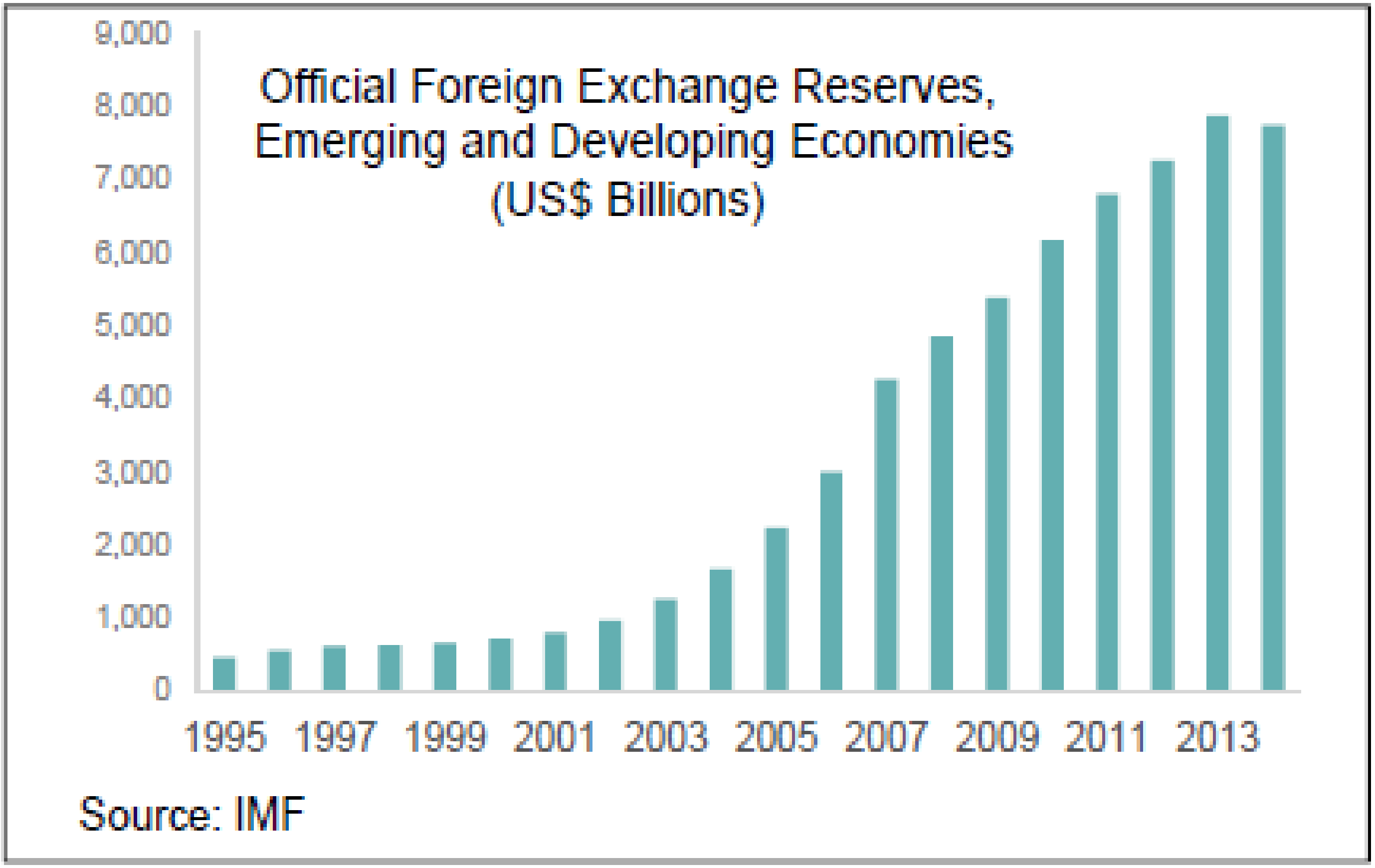
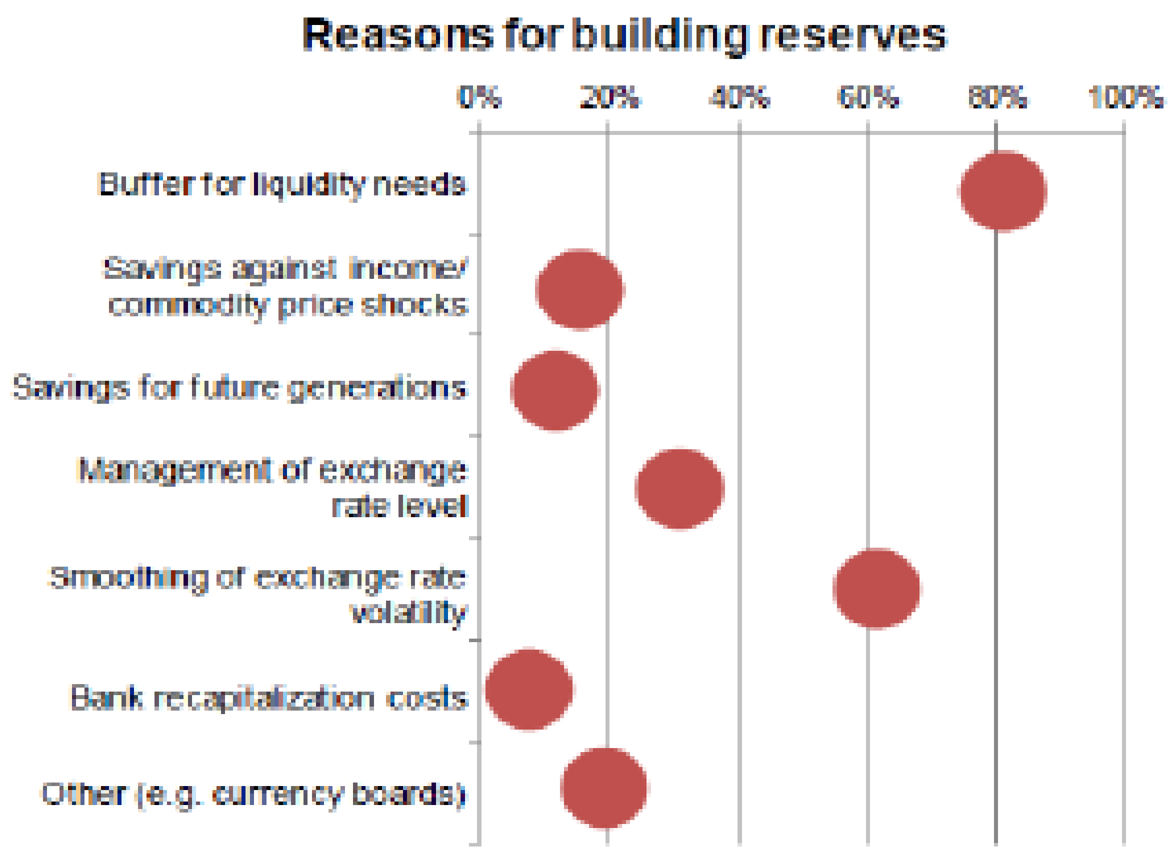
- Debates, while connected, often occur in parallel



From : Bunda (2016)



Source: IMF survey of reserve managers.

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- The accumulation of reserves may change private incentives and lead to increased borrowing...
- ...making liquidity more scarce if there is a crisis: is the strategy self defeating?

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- What are the determinants of optimal reserves?
- How do they compare with other tools (e.g. macroprudential)?

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- International reserves enable the central bank to provide international liquidity and alleviate financial constraints when they bind
- Reserves accumulation does provide incentives for private borrowing
- The optimal level of reserves is tightly linked to the impact and nature of *ex post* intervention

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5. ...and on the specific policies that the central bank use in the event of a liquidity crunch
6. An increase in ex ante *uncertainty* also justifies a buildup of reserves

Related Literature

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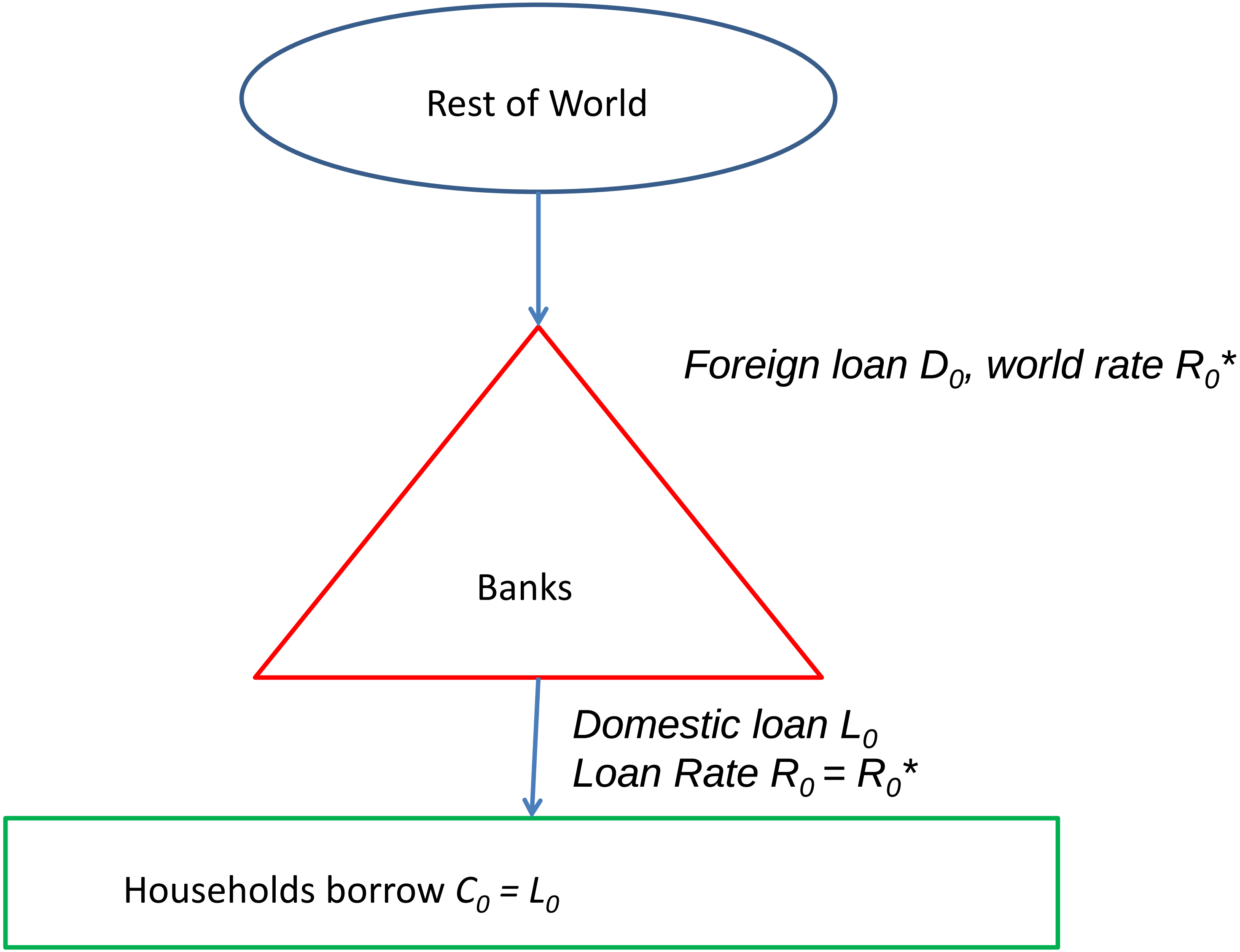
- *Macroprudential policy*: Benigno, Chen, Otrok, Rebucci, and Young (2013), Jeanne and Korinek (2017), Korinek and Simsek (2016), Schmitt Grohe and Uribe (2018, 2021)

A Basic Model



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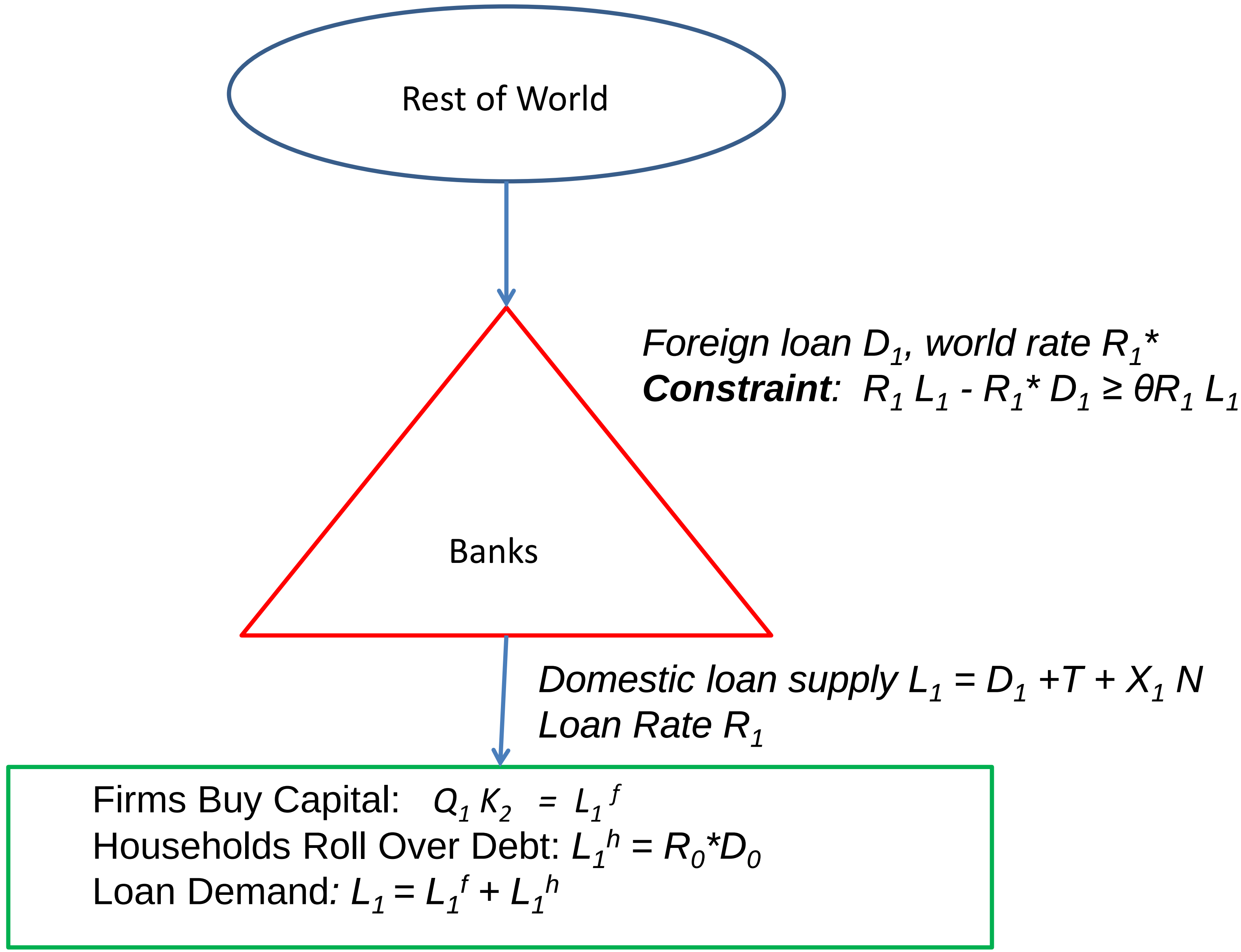

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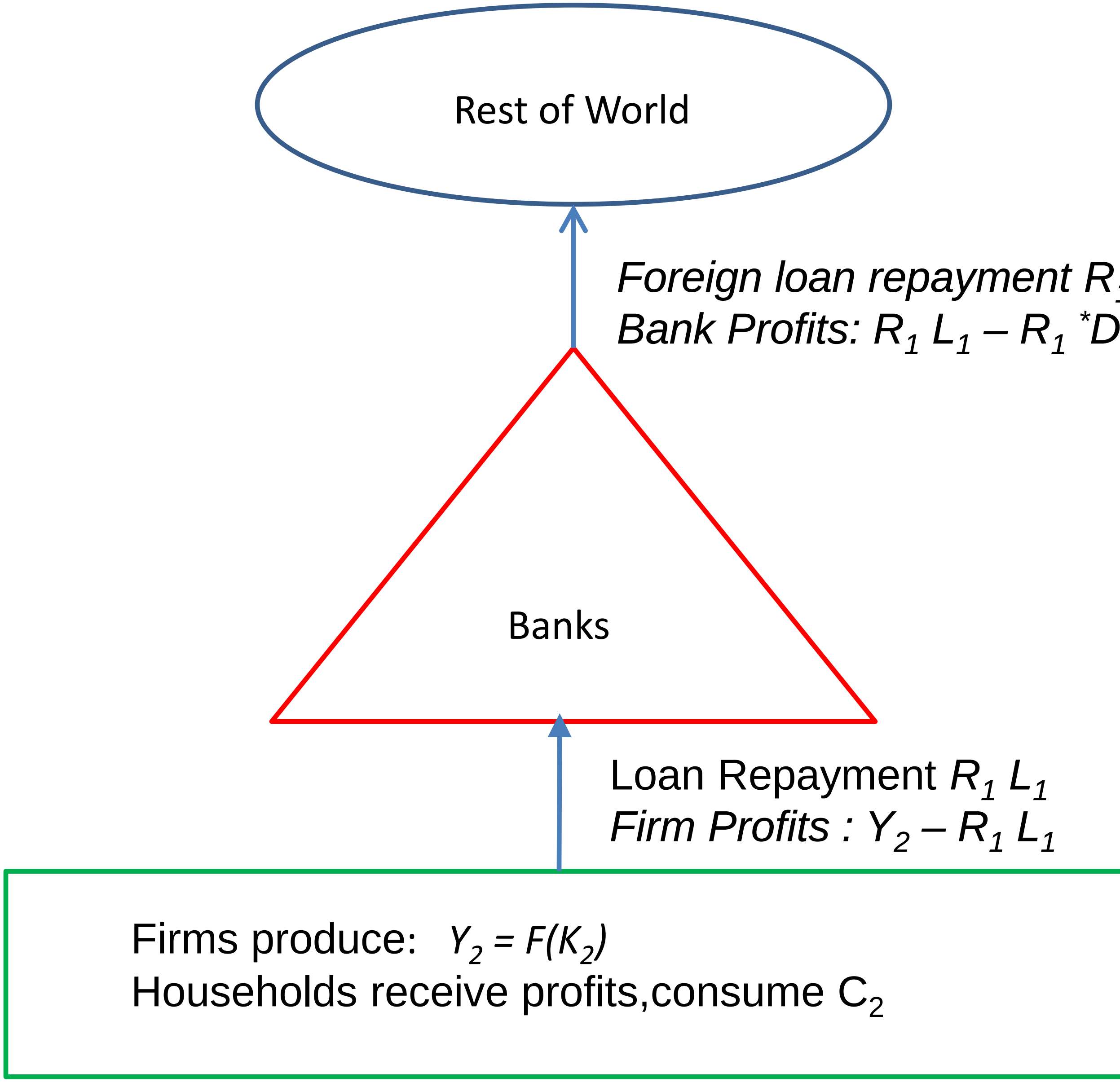

Repayment Period

Basic Model

- $t = 0, 1, 2$
- Small open economy
- Two goods: tradables (numeraire) and non tradables
- Domestic households and firms borrow from rest of the world via financial intermediaries (banks)
- Financial intermediation subject to frictions and shocks

Households

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- (Uncertainty is realized at $t = 1$, so R_1 is random)
- $t = 2$:

$$\begin{aligned} C_2 &= \Pi^b + \Pi^f - R_1 L_1^h \\ &= \Pi^b + \Pi^f - R_1 R_0^* C_0 \end{aligned}$$

Initial consumption (and debt) are then given by the first order condition:

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$==>$ Note that if $ER_1 > R_1^*$, borrowing is inefficiently low

Production and Investment

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- Profits are then $\Pi^f = AK_2^{\alpha} - R_1 Q_1 K_2$, so that the demand for capital is given by:

$$\alpha AK_2^{\alpha - 1} = R_1 Q_1$$

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- These eqs and $\alpha A K_2^{\alpha-1} = R_1 Q_1$ determine I_W, K_2, Q_1, and X_1, given R_1

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- $t = 2$: Bank profits are

$$\Pi^b = R_1 L_1 - R_1^* D_1$$

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- θ can take n values, denoted by θ_s, $s = 1, \ldots n$, each with probability $\pi_s > 0$

Laissez Faire Equilibrium

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- If $R_1 > R_1^*$, the bank borrows as much as it can, and lends

$$L_1 = \frac{1}{1 - (1 - \theta)\phi}(T + X_1 N)$$

where $\phi = R_1 / R_1^*$ is the interest rate spread.

If financial constraints do **not** bind, $R_1 = R_1^*$, and all other variables take their frictionless (**f**) values:

$$\alpha A K_{2f}^{\alpha-1} = R_1^* Q_{1f} = R_1^* X_{1f}^{\gamma}$$

$$\frac{X_{1f} N}{I_{wf}} = \frac{\gamma}{1-\gamma}$$

$$K_{2f} = \kappa N^{\gamma} I_{wf}^{1-\gamma}$$

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- The threshold $\vec{\theta}$ is **endogenous** and, in particular, falls with C_0

If $\theta > \vec{\theta}$, then $R_1 > R_1^*$ and relative prices adjust to clear markets.

The equilibrium exchange rate then solves:

$$R_0^* C_0 + Q_1 K_2 = \frac{1}{1-(1-\theta)\phi}(T + X_1 N)$$

where the spread ϕ is given by

$$\phi = R_1 / R_1^* = \left(\frac{X_f}{X_1}\right)^{\gamma+(1-\alpha)(1-\gamma)}$$

Full Equilibrium

Recall that, in any continuation equilibrium

$$\begin{aligned} R_1 &= R_1^* \text{ if } \theta \leq \vec{\theta} \\ &= \rho(C_0, \theta) \text{ if } \theta > \vec{\theta} \end{aligned}$$

For equilibrium, C_0 must then satisfy:

$$U'(C_0) = \beta R_0^* \left[R_1^* F(\vec{\theta}) + \sum_{\theta_s > \vec{\theta}} \rho(C_0, \theta) \pi_s \right]$$

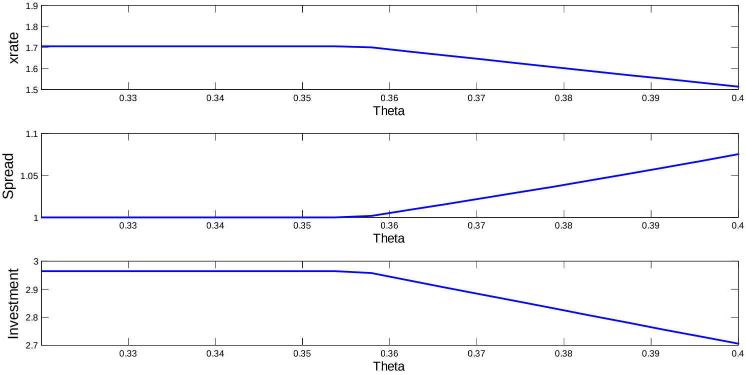
where

$$F(\vec{\theta}) = \sum_{\theta_s \leq \vec{\theta}} \pi_s$$

is the probability of no crisis.

Equilibrium Implications

Laissez Faire: Continuation Equilibrium



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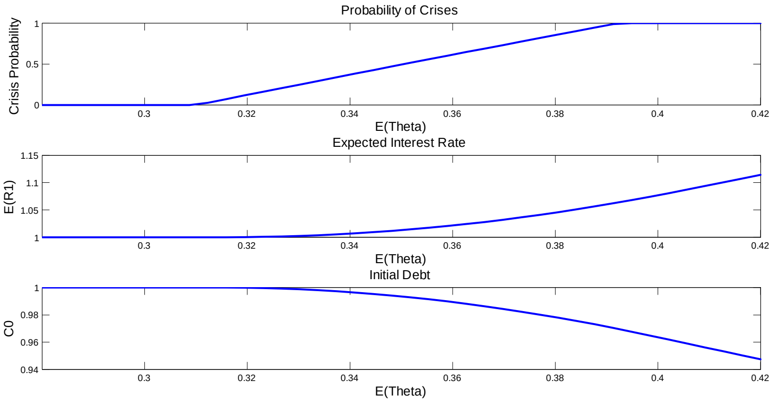
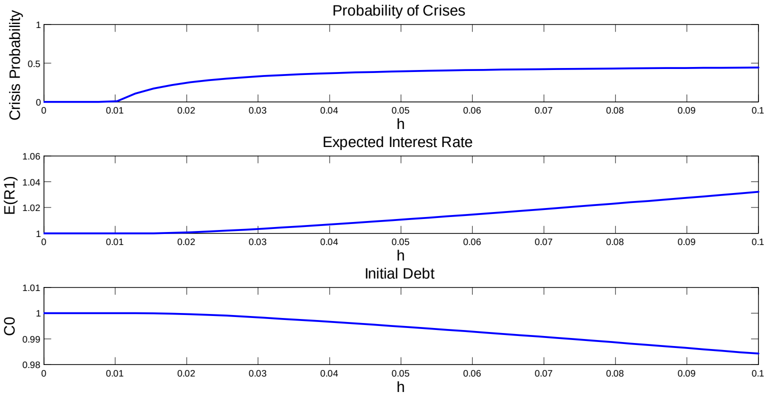
- Some determinants are "obvious": i.e. lower productivity (lower A) lead to lower $\hat{\theta}$ and higher probability of crises
- Other ones are novel e.g. an increase in uncertainty can lead to higher crises probability



Laissez Faire and $E(\theta)$



Uncertainty and Equilibrium

First Best Allocation and Inefficiencies

The First Best Problem

The *first best* problem maximizes the representative household's welfare subject only to the resource constraints:

$$
\begin{aligned}
C_2 &= AK_2^{\alpha} - R_0^* R_1^* C_0 - R_1^*(I_W - T) \\
K_2 &= \kappa N^{\gamma} I_W^{1-\gamma}
\end{aligned}
$$

Consumption and Investment Inefficiencies

The first best solution requires:

$$U'(\hat{C}_0) = \beta R_0^* R_1^*$$

$$\alpha A \hat{K}_2^{\alpha-1} = R_1^* [1/(1-\gamma)\kappa N^\gamma \hat{I}_W^{-\gamma}]$$

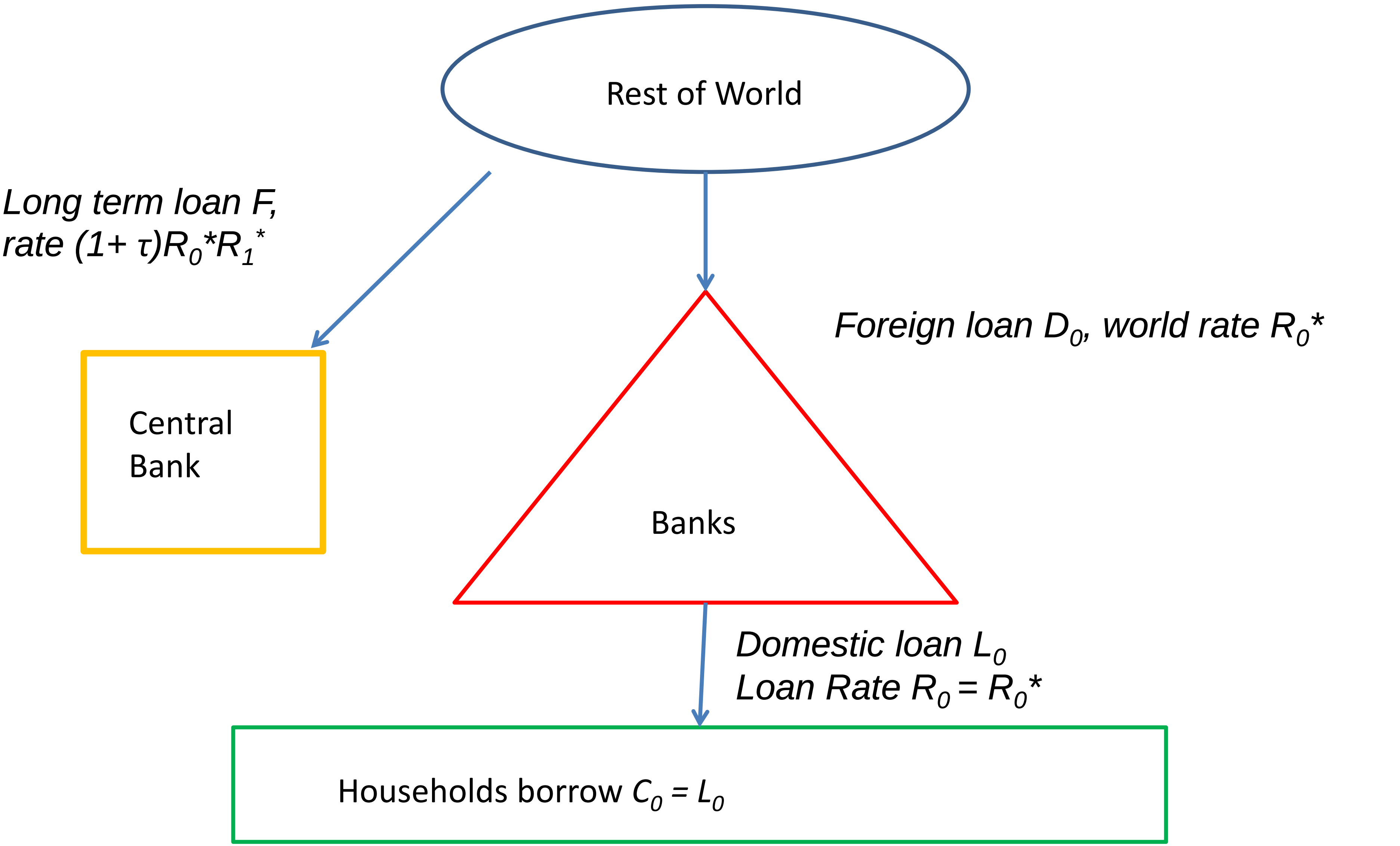
while under laissez faire:

$$U'(C_0) = \beta R_0^* E(R_1)$$

$$\alpha A K_{2s}^{\alpha-1} = R_{1s} [1/(1-\gamma)\kappa N^\gamma I_{Ws}^{-\gamma}],\ s = 1, ...n$$

$==>$ Both investment and initial consumption under laissez faire are lower than first best

FX Reserves and Intervention



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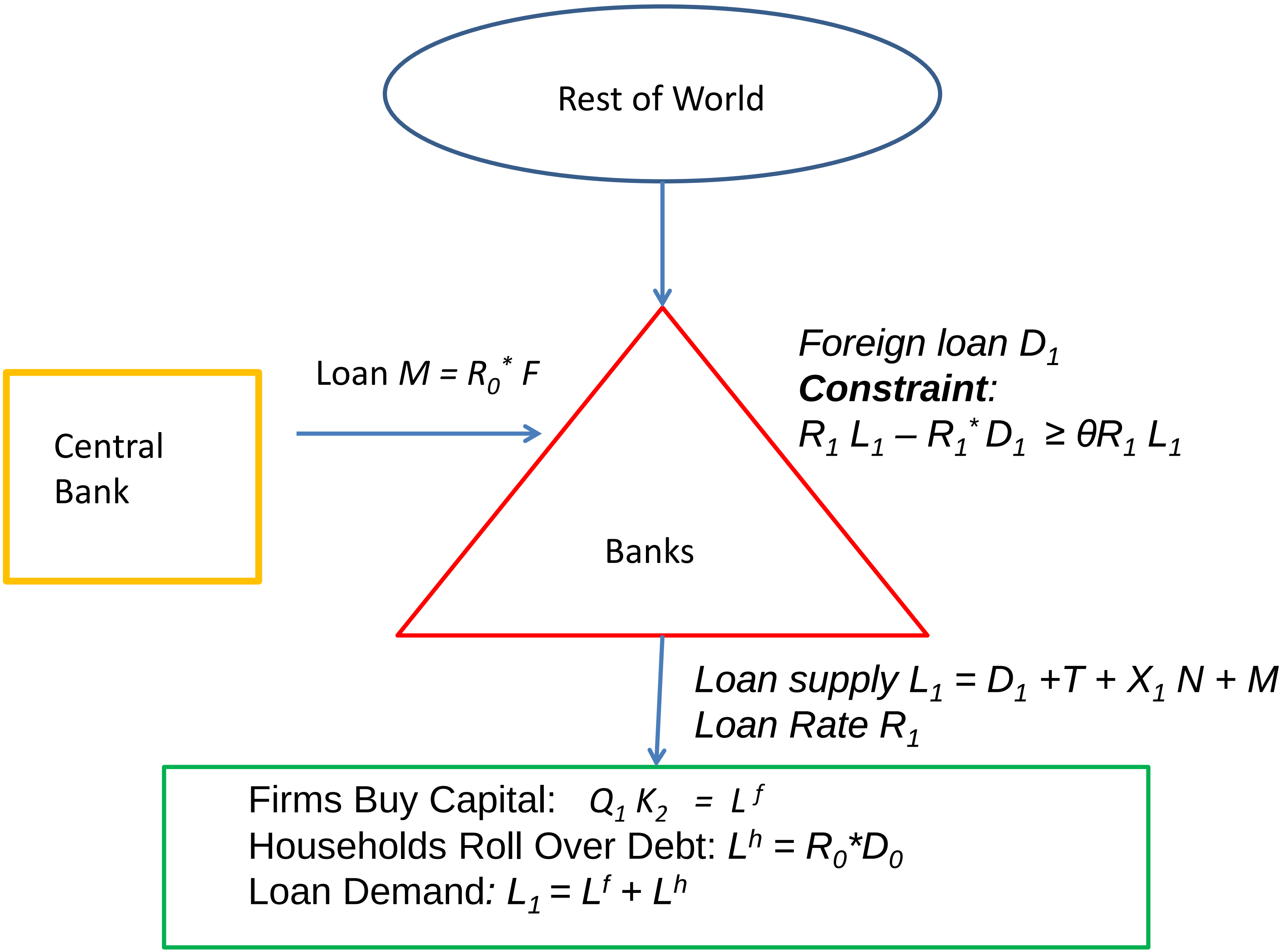

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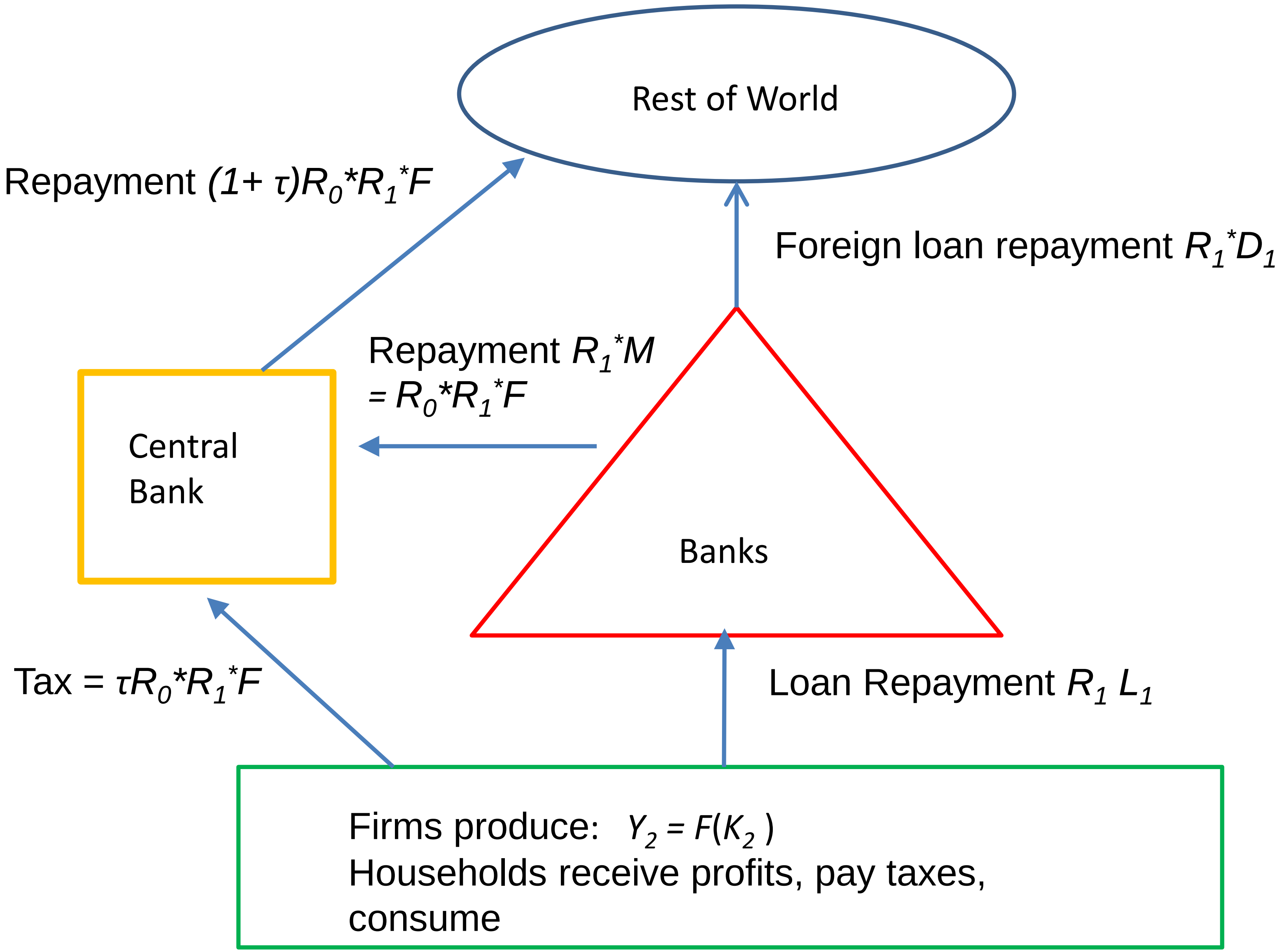

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- The central bank can invest F short term in the world market, earning R_0^* and then R_1^*
- But in period $t = 1$ it also has the option to use $R_0^* F$ to enact policies aimed at alleviating financial frictions, if these turn out to be binding.
- We assume that the central bank cannot borrow (more) abroad at $t = 1$.

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- This justifies active central bank policy.

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- We call this a **liquidity policy with F reserves**
- In terms of Gertler-Kiyotaki (2011), the central bank provides "liquidity facilities"
- For the analysis, we assume that crises occur with positive probability in laissez faire.

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$$R_1 L_1 - R_1^* (D_1 + R_0^* F) \geq \theta R_1 L_1 - R_1^* R_0^* F$$

i.e. it reduces to the same one as in laissez faire

- But loan supply is now constrained by

$$L_1 \leq \frac{1}{1 - (1-\theta)\phi} (T + X_1 N + R_0^* F)$$

Equilibrium with Liquidity/Reserves Policy

$$U'(C_0) = \beta R_0^* \sum_s \pi_s R_{1s}$$

$$C_{2s} = AK_{2s}^{\alpha} - R_1^* I_{Ws} + R_1^* T - R_0^* R_1^* C_0 - \tau R_0^* R_1^* F$$

$$\alpha AK_{2s}^{\alpha-1} = R_{1s} Q_{1s}$$

$$K_{2s} = \kappa N^{\gamma} I_{Ws}^{1-\gamma}$$

$$Q_{1s} = X_{1s}^{\gamma}$$

$$I_{ws} = (1-\gamma) Q_{1s} K_{1s}$$

$$\frac{R_1^*}{R_1^* - (1-\theta_s) R_{1s}} [T + X_{1s} N + R_0^* F] - (R_0^* C_0 + Q_{1s} K_{1s}) \geq 0$$

with if $R_{1s} > R_1^*$

Two Useful Lemmas

Lemma 1. There is $\bar{F}$ such that for any $F \geq \bar{F}$, financial frictions do not bind in the competitive equilibrium with an F reserves policy.

Lemma 2. Given $R_{1s} \geq R_1^*$, there are unique $I_{ws}, K_{2s}, X_{1s}, Q_{1s}$,that satisfy the associated competitive equilibrium conditions (for any C_0, F in $[0, \bar{F}]$)

We can now write C_{2s} as a function of R_{1s}, C_0, and F :

$$C_{2s} = C_2(R_{1s}, C_0, F) = AK_{2s}^{\alpha} - R_1^* I_{Ws} + R_1^* T - R_0^* R_1^* C_0 - \tau R_0^* R_1^* F$$

where, in the RHS, K_{2s} and I_{Ws} are seen as the functions of R_{1s} in Lemma 2.

Also, the borrowing constraint can be written as:

$$\Psi(R_{1s}, C_0, F) \geq 0, \;\; = \;\text{if } R_{1s} > R_1^*$$

Optimal (Second Best) Policy

A Second Best Problem

The associated **second best problem** is now to choose $C_0 \geq 0$, $F \geq 0$, and $R_{1s} \geq R_1^*$, $s = 1, ...n$ to maximize

$$U(C_0) + \beta \sum_s \pi_s C_2(R_{1s}, C_0, F)$$

subject to

$$U'(C_0) = \beta R_0^* \sum_s \pi_s R_{1s}$$

and

$$\Psi(R_{1s}, C_0, F) \geq 0, \quad = \text{ if } R_{1s} > R_1^*$$

Optimal Reserves

The FOC wrt F gives:

$$\sum_s \beta \pi_s \omega_s \left[\frac{R_1^*}{R_1^* - (1-\theta_s) R_{1s}} \right] R_0^* \leq \tau R_0^* R_1^*, \quad = \text{ if } F > 0$$

Proposition: (i) If $\tau = 0$, optimal liquidity policy with reserves prescribes $F \geq \bar{F}$, so that crises do not occur.

(ii) Let Δ denote the **laissez faire** value of the LHS of the preceding inequality. Then, $F = 0$ is optimal only if $\tau \geq \Delta / R_0^* R_1^*$

(iii) If $0 < \tau < \Delta / R_0^* R_1^*$, the optimal liquidity policy with reserves implies $0 < F < \bar{F}$, which allows crises to occur with positive probability.

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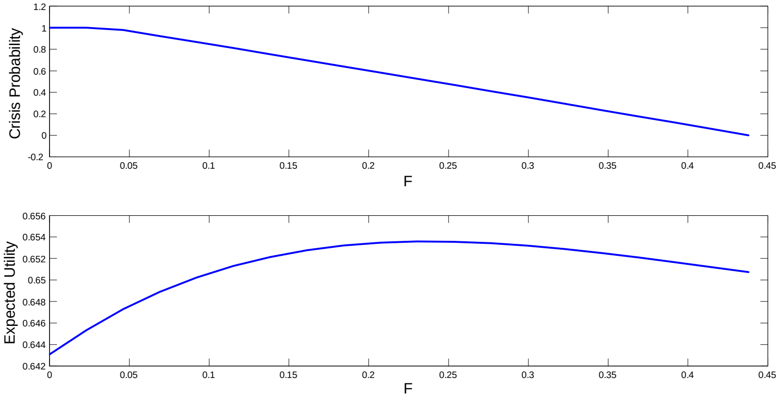
2. If τ is too large, it is optimal not to intervene. This is because the benefits from policy are bounded.
3. For intermediate values of τ, some reserves accumulation and liquidity provision is warranted, but crises are allowed to occur with positive probability.



Reserves, Crisis Probability, and Utility

Intuition

The expected value of choosing F is

$$\begin{aligned} V(F) &= U(C_0) + \beta E C_2 \\ &= U(C_0) - \beta R_1^* R_0^* C_0 + \beta E\left[A K_2^{\alpha} - R_1^*(I_w - T)\right] - \beta \tau R_0^* R_1^* F \end{aligned}$$

with

$$V'(F) = \left[U'(C_0) - \beta R_1^* R_0^*\right] \frac{dC_0}{dF} + \beta E\left\{(R_1 - R_1^*) \frac{dI_w}{dF}\right\} - \beta \tau R_0^* R_1^*$$

- If $\tau = 0$, holding reserves has no opportunity cost

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- The marginal gain to eliminating crises completely is of second order, so it is not optimal to do that if $\tau > 0$

Liquidity, Reserves, and Macroprudential Tools

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- Suppose the planner can add a macroprudential policy that controls the initial debt and consumption C_0.

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Adding Macroprudential Tools

- Suppose the planner can add a macroprudential policy that controls the initial debt and consumption C_0.
- If $\tau = 0$, macroprudential policy is superfluous.
- Macroprudential policy always faces a trade-off: it can reduce inefficiency in initial consumption only by increasing inefficiency in investment.
- If $\tau > 0$ but small, the financial constraint must bind with positive probability, as before. Hence it is (second best) optimal to use both kinds of policies to reduce inefficiencies **but not** completely erase them.

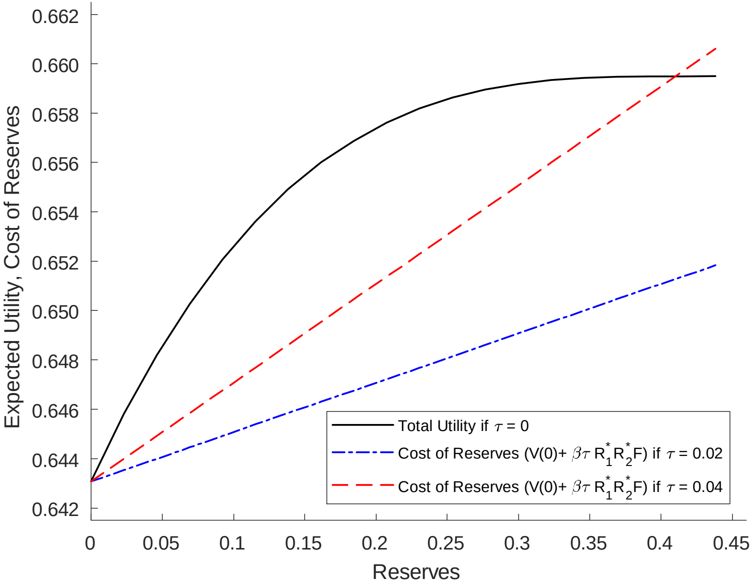
Determinants of Optimal Reserves

Parameter Values

$$
\begin{aligned}
R^* &= 1 \\
\eta &= 1.4 \\
\gamma &= 0.5 \\
\alpha &= 0.8 \\
\theta &\in [0.36, 0.44] \\
\tau &= 0.02 \\
\sigma &= 2
\end{aligned}
$$

The Cost of Reserves

Here the relevant cost is the term premium τ



0.662
0.660
0.658
0.656
0.654
0.652
0.650
0.648
0.646
0.644
0.642
Expected Utility, Cost of Reserves
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
Reserves
Total Utility if $\tau = 0$
Cost of Reserves (V(0)+ $\beta\tau R_1^* R_2^* F$) if $\tau = 0.02$
Cost of Reserves (V(0)+ $\beta\tau R_1^* R_2^* F$) if $\tau = 0.04$

Reserves and Financial Development

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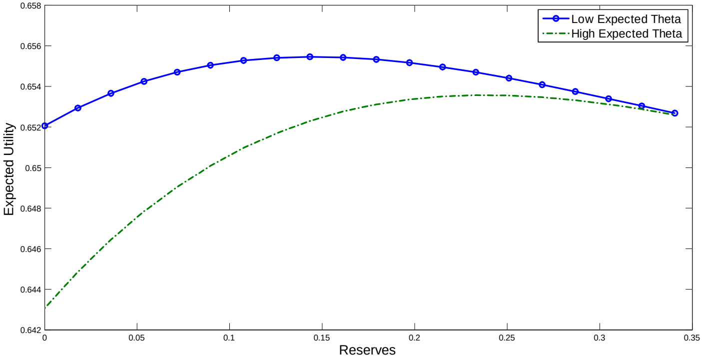
- This may capture differences in financial development
- Correspondingly, one would expect that optimal reserves should be smaller



Reserves and $E(\theta)$

Optimal Reserves and Uncertainty

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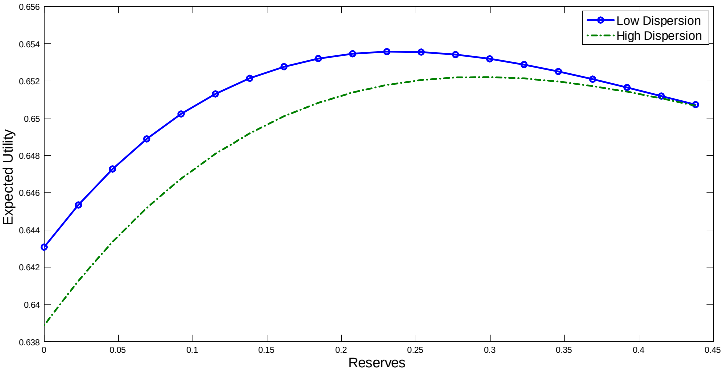
- A mean preserving spread of θ leads to higher reserves

Optimal Reserves and Uncertainty

- A mean preserving spread of θ leads to higher reserves
- This is in line with intuition, and with observed experiences



Uncertainty and Optimal Reserves

Reserves Accumulation and Ex Post Policy

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- As in CCV, the central bank uses reserves more effectively if it lends them to banks instead of firms or households in a credit crunch

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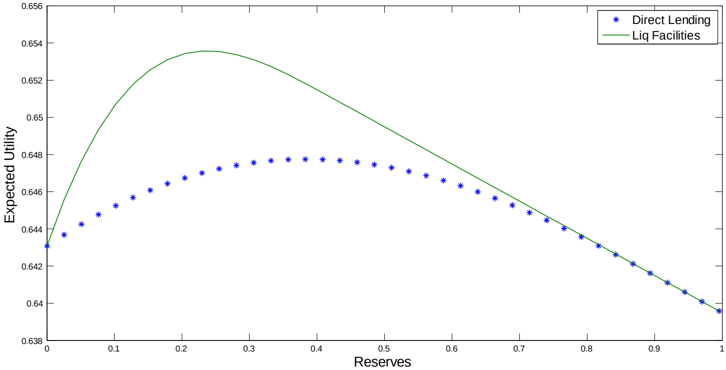
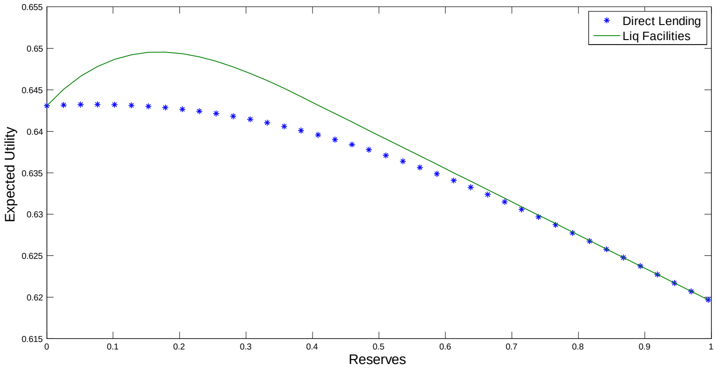
Reserves Accumulation and Ex Post Policy

- As in CCV, the central bank uses reserves more effectively if it lends them to banks instead of firms or households in a credit crunch
- But **direct lending** may be more feasible because of other reasons (e.g. political)
- Must optimal reserves be larger with direct lending?



Expected Utility, Reserves, and Ex Post Policy



Same, but with $\tau = 0.04$

Final Remarks

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- Interesting directions: dollarization; dynamics; more on policy options

Thank You!!