Learning Objectives:
(**Review**) graphs of exp/log functions, simplifying expressions with exp/log, solving equations with exp/log, exponential growth and decay, special values of trig functions, solving basic equations with trig, setting up optimization problems

On the final exam, students must know all of the following formulas and special values, and similar kinds of formulas and special values (for instance, the total surface area of a rectangular box with an open top). If necessary, the formula for the volume or the surface area of a sphere, or the volume or the surface area of a cylinder, would be supplied.

- the exact values of the 6 trigonometric functions at the standard special angles
- standard values of logarithmic and exponential functions:
  e.g., ln(1) and 3^0
- quadratic formula
- Pythagorean theorem
  (also the 3-4-5, 6-8-10, and 5-12-13 triangles to save time)
- area of a triangle (given base and height)
- area and perimeter of a rectangle (including a square)
- area and circumference of a circle
- volume and total surface area of a rectangular solid (including a cube)

Precalculus II Review

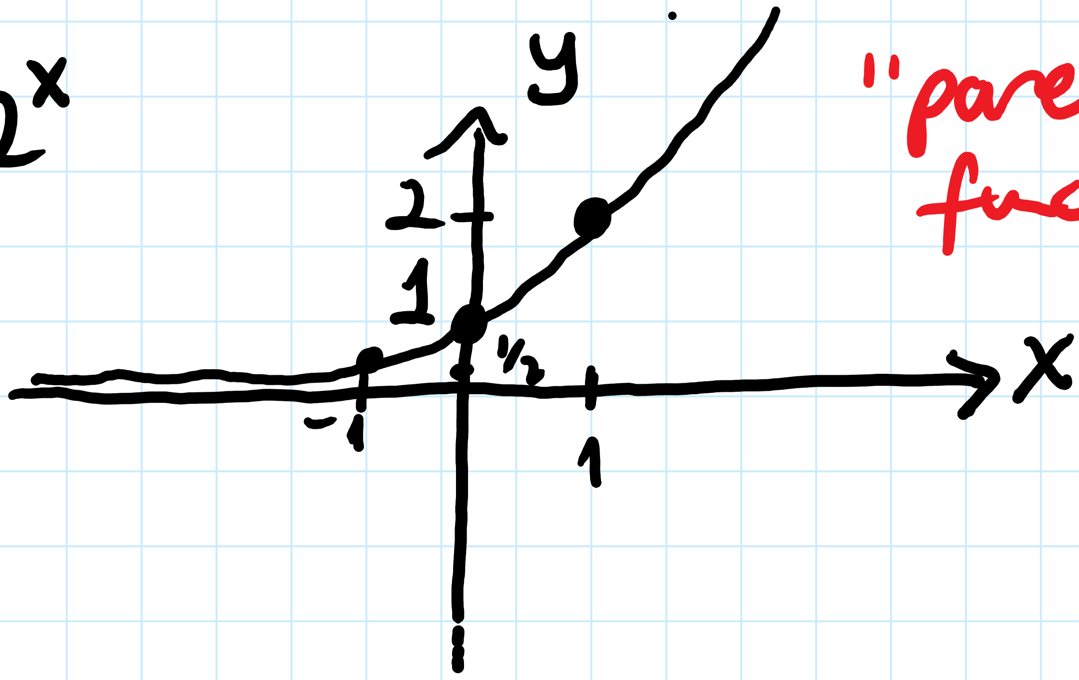
Exponential Function: The function f is an exponential function if $f(x) = b^x$

where b is a pos. constant other than 1, and x is a real number.

e.g.: Graph of $f(x) = 2^x$

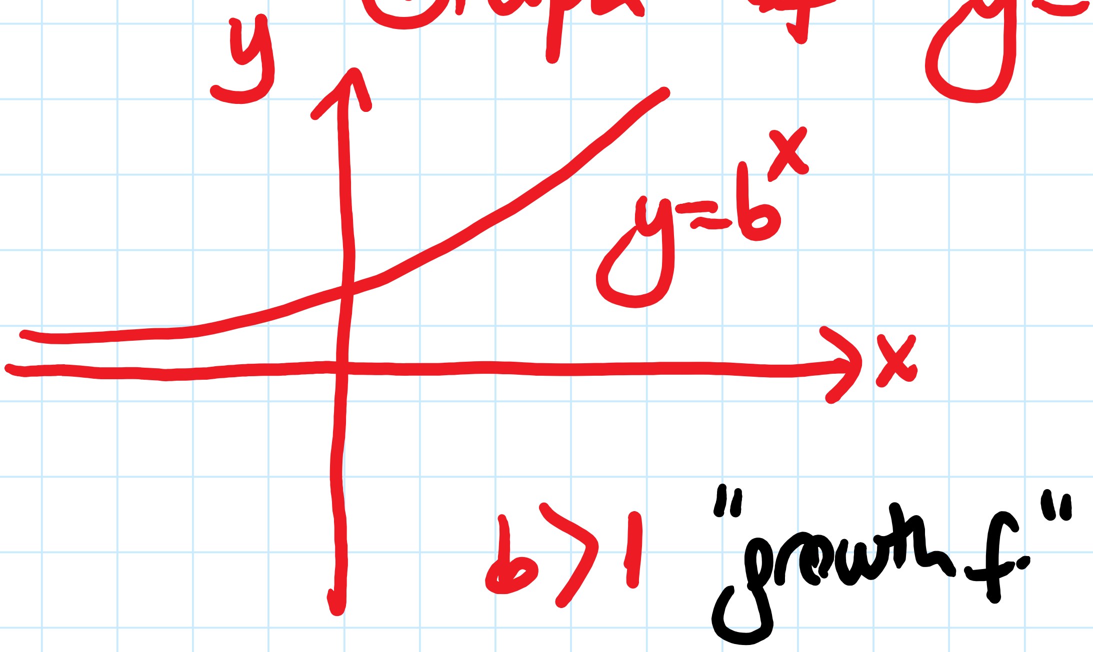
x	$f(x)$
0	1
1	$2^1 = 2$
-1	$2^{-1} = \frac{1}{2}$

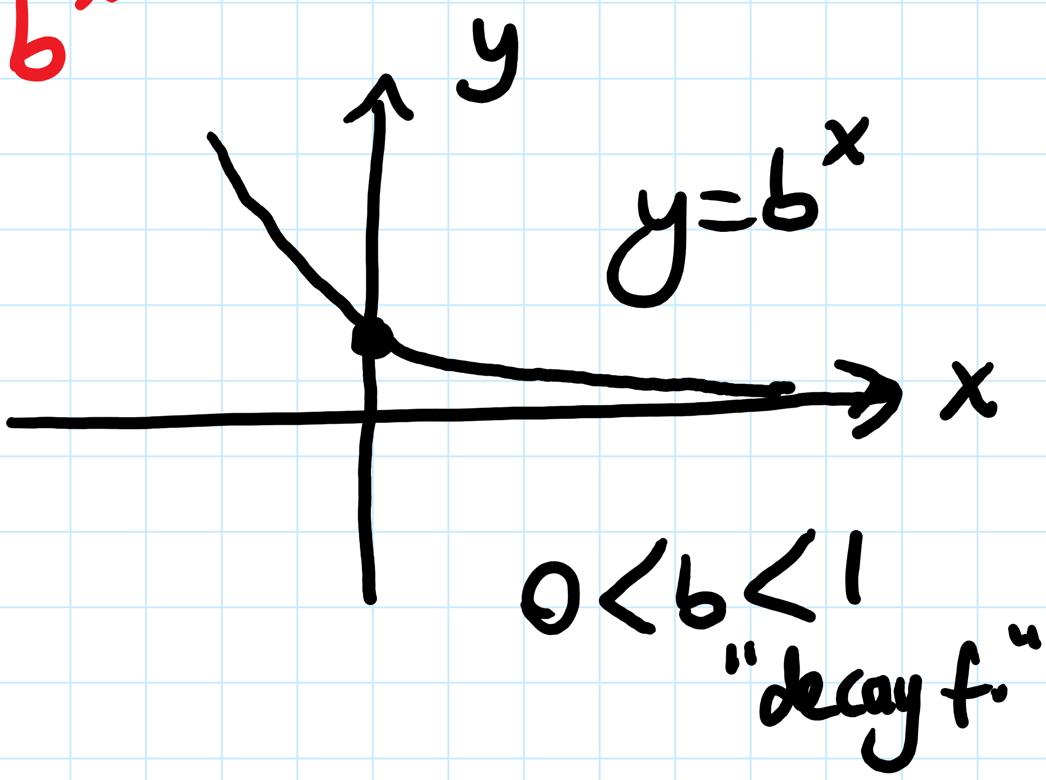
(0,1)



"parent functions"

Graph of $y=b^x$





Page 110 → basic prop. of expo. func.

e.g: $2^{x^2+3}=16=2^4 \Rightarrow \cancel{2}^{x^2+3}=\cancel{2}^4$

$(x^2-y^2=(x-y)\cdot(x+y))$

$x^2+3=4$

$x^2-1=0=(x-1)(x+1)$

$\boxed{x=\mp 1}$

E.g: $2^x \cdot 3^{x+1}=108$

Friday, September 6, 2019
9:10 AM

$\dfrac{2^x \cdot 3^x \cdot \cancel{3^1}}{\cancel{3}}=\dfrac{108}{3} \Rightarrow 2^x \cdot 3^x = 36$

$(2\cdot 3)^x=36$

$6^x=36 \Rightarrow x=2$

E.g. $(\sqrt{2})^{x^2} = \frac{8^x}{4} \Rightarrow (2^{1/2})^{x^2} = \frac{(2^3)^x}{2^2}$

$[\sqrt[2]{x} = x^{\frac{1}{2}}]$

$[\sqrt[3]{x} = x^{\frac{1}{3}}]$

$[\sqrt[3]{x^2} = x^{2/3}]$

$2^{\frac{1}{2}\cdot x^2} = \frac{2^{3x}}{2^2}$

$\cancel{2}^{\frac{x^2}{2}} = \cancel{2}^{3x-2}$

$\frac{x^2}{2} = \frac{3x-2}{1}$

$x^2 = 6x - 4$

$x^2 - 6x + 4 = 0$

$a = 1,\ b = -6,\ c = 4$

Quadratic Formula:

$ax^2 + bx + c = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{6 \pm \sqrt{(-6)^2 - 4\cdot 1\cdot 4}}{2\cdot 1}$

$3 + \sqrt{5}$

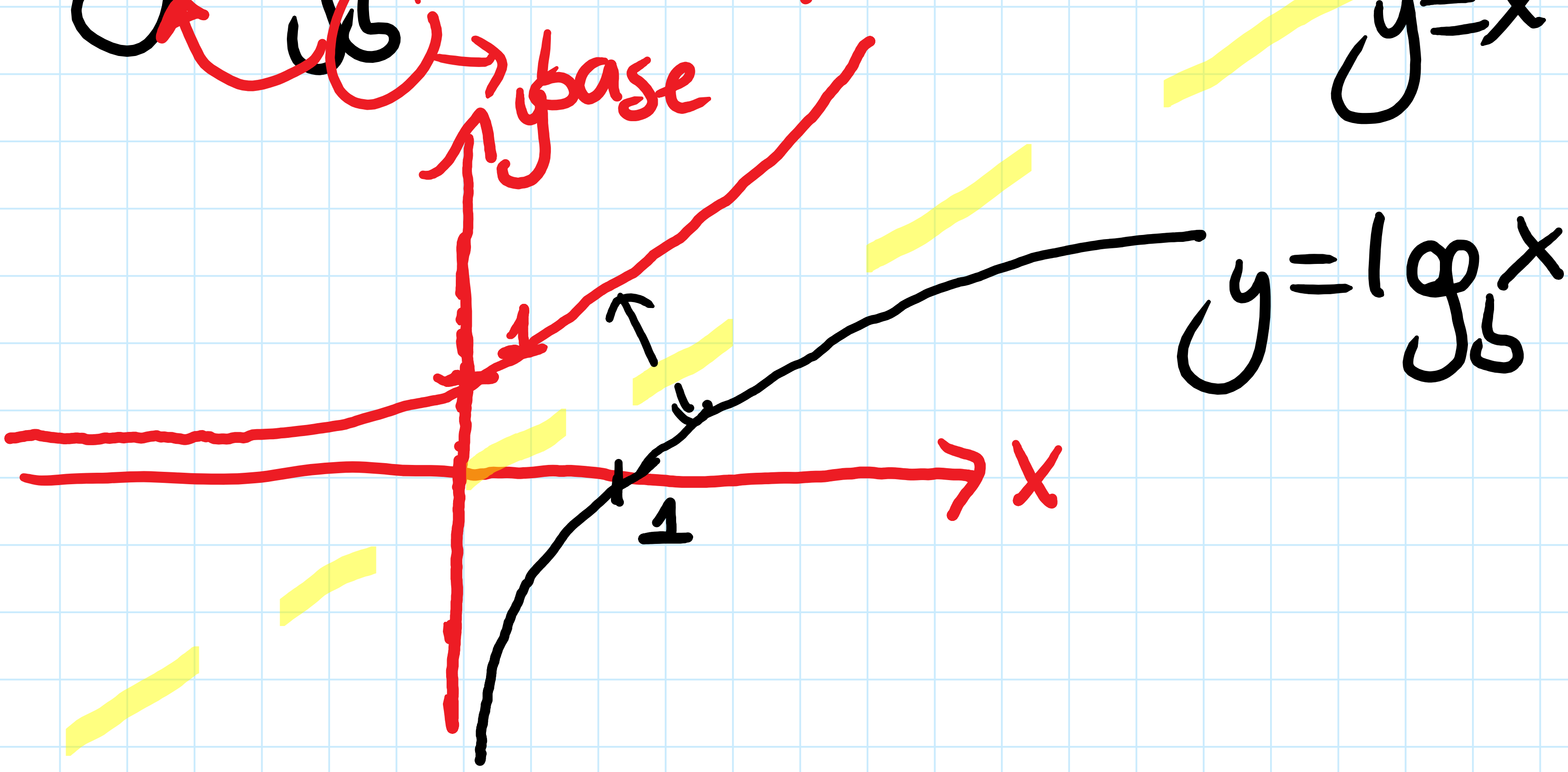
$3 - \sqrt{5}$

$\boxed{3 \pm \sqrt{5}} = \frac{6 \pm 2\sqrt{5}}{2} = \frac{6 \pm \sqrt{20}}{2}$

Logarithmic Functions

Def: If $b > 0$, $b \neq 1$; the logarithm of x to the base b is the function $y = \log_b x$ that satisfies $b^y = x$ ($b^y > 0$ for all y)

$y = \log_b x \Rightarrow b^y = x$



Some properties of logarithmic functions ($b > 0$ and $b \neq 1$)

Product Rule $\quad \log_b(x \cdot y) = \log_b x + \log_b y$

Quotient Rule $\quad \log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

Power Rule $\quad \log_b x^p = p \cdot \log_b x$

Page 112

Evaluate & simplify

E.g: $\log_2 \frac{1}{8} + \log_2 128 = \log_2 2^{-3} + \log_2 2^7$

$\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$

$= -3 \cdot \cancel{\log_2 2}^{1} + 7\cancel{\log_2 2}^{1} = -3+7 = \boxed{4}$

$128 = 2^7$

E.g: Solve the eq. $\log_3 (2x+1) - 2\log_3 (x-3) = 2$

$[\log x - \log y = \log\left(\frac{x}{y}\right)]$

$\log_3 (2x+1) - \log_3 (x-3)^2 = 2$

$\log_3 \frac{2x+1}{(x-3)^2} = 2 \Rightarrow \frac{3^2}{1} = \frac{2x+1}{(x-3)^2}$

verify

$\boxed{x=4}$

$9(x-3)^2 = 2x+1 \Rightarrow 9(x^2-6x+9) = 2x+1$

$9x^2 - 54x + 81 - 2x - 1 = 0$

$9x^2 - 56x + 80 = 0 = (9x-20)(x-4)$

$x = \frac{20}{9}, 4$

CMSA

$9x \quad -20$

$x \quad -4$

$-36x - 20x = -56x$ ✓

Natural Base e

$e \approx 2.7182818284 5 \ldots$

natural exponential base, and can be def. by a limit.

$$e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$

"Euler's number"

EXP NOTATION: $\exp(f(x))$ means $e^{f(x)}$

Common Logarithm: $\log_{10} x = \log x$

Natural Logarithm: $\log_e x = \ln x$

Theorem 2.9

1) $\ln 1 = 0 \Rightarrow e^0 = 1$

2) $\ln e = 1 \Rightarrow e^1 = e$

3) $e^{\ln x} = x$ for all $x > 0$

4) $\ln e^y = y$ for all y

5) $b^x = e^{x \cdot \ln b}$ for any $b > 0$, $b \neq 1$

Exp. Solve: $6^x = 200$

$$\ln 6^{x} = \ln 200$$

$$\frac{x \cdot \ln 6}{\ln 6} = \frac{\ln 200}{\ln 6}$$

$$\boxed{x = \frac{\ln 200}{\ln 6}}$$

E.g: Solve $\dfrac{e^{x^2}}{e^{x+6}} = 1$

$$e^{x^2-(x+6)} = e^{x^2-x-6} = 1$$

$$\boxed{e^0 = 1}$$

$$\cancel{e}^{x^2-x-6} = \cancel{e}^0$$

$$x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0$$

$-3 \quad 2$

$$\boxed{x = 3,\ x = -2}$$

TRIGONOMETRY REVIEW

Special values of trigonometric function

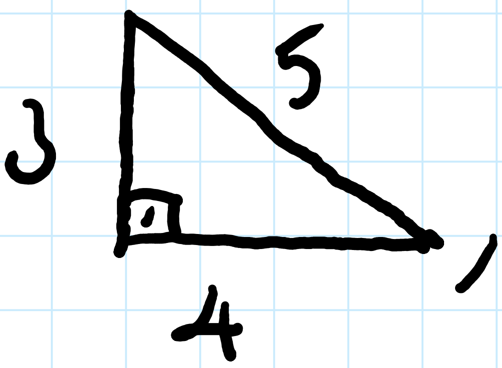
Angle (θ)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\frac{\sin\theta}{\cos\theta} = \tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef.	0	undefined
$\sec\theta$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	undef.	-1	undefined
$\csc\theta$	undef.	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	undef.	-1
$\cot\theta$	undef.	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	undef.	0

Trig. Identities:

$\sec\theta = \frac{1}{\cos\theta}$, $\csc\theta = \frac{1}{\sin\theta}$, $\cot\theta = \frac{1}{\tan\theta}$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin^2\theta + \cos^2\theta = 1$

Special Right Triangles



, 6-8-10 . . .

(multiples of 3-4-5 Δs)

5-12-13

10-24-26

⋮

Solving Basic Eq. w/ Trig.

E.g: Solve the eq. $3\sin x - 2 = 5\sin x - 1$; $[0, 2\pi]$

$$\cancel{3\sin x} - 2 = 5\sin x - 1$$
$$\cancel{-3\sin x} \qquad -3\sin x$$
$$-2 = 2\sin x - 1$$
$$+1 \qquad\qquad +1$$
$$-\frac{1}{2} = \sin x \Rightarrow \sin x \text{ is neg. in QIII, QIV}$$

$(x, y) \to (\cos\theta, \sin\theta)$

$$\Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

E.g: Solve the eq: $2\cos^2\theta + \cos\theta - 1 = 0$; $[0, 2\pi]$

$$2\cos^2\theta + \cos\theta - 1 = 0$$

$2\cos\theta \qquad -1$

$\cos\theta \qquad 1$

CMJA

$2\cos\theta - \cos\theta = \cos\theta$ ✓

$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$

$$\cos\theta = \frac{1}{2}; \quad \cos\theta = -1$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}; \quad \theta = \pi$$

Setting up an optimization problem:

- E.g: A carpenter needs to build a rectangular box by following these specifics:
  - width of the box should be half of the length (l)
  - height of the box should be three times the width

a) Write an expression for volume ($V(l)$)

b) Write an expression for surface area ($A(l)$)

c) If the sum of the box's height and length is at most 5 units, then what would be the domain of $A(l)$?

<u>Formulas</u>

$V = l \cdot w \cdot h$

$A = 2 \cdot (lw + lh + wh)$

<u>Given</u>

$w = \frac{l}{2}$, $h = 3 \cdot w$

Dimensions: l, $w = \frac{l}{2}$, $h = 3w = 3 \cdot \frac{l}{2}$

a) $V(\ell) = \ell \cdot \frac{\ell}{2} \cdot \frac{3\ell}{2} = \frac{3\ell^3}{4}$

b) $A(\ell) = 2\left(\ell \cdot \frac{\ell}{2} + \ell \cdot \frac{3\ell}{2} + \frac{\ell}{2} \cdot \frac{3\ell}{2}\right)$

$= 2\left(\frac{\ell^2}{2_{(2)}} + \frac{3\ell^2}{2_{(2)}} + \frac{3\ell^2}{4}\right) = 2 \cdot \frac{11\ell^2}{4} = \boxed{\frac{11\ell^2}{2}\ \text{unit}^2}$

c) $\underbrace{h+\ell}$ $\leq$ 5 units

$\frac{3\ell}{2} + \ell \leq 5 \Rightarrow \frac{5\ell}{2} \leq 5 \Rightarrow \ell \leq 2$

since ℓ can not be a negative number, therefore; domain of $A(\ell)$ is $[0,2]$.