

Math 135: Fall 2019

Instructions:

Please show all your work in order to receive proper credit. You are not allowed to use any calculator, formula sheet, notes or electronic devices. Quiz should be completed in one seating with no breaks. All final answers should be in the simplest form. Box your final answer.

Problem 1. (4 points) Compute the limit:

$$\lim_{x \rightarrow 0} \frac{x^2 - 2x + \sin x}{x}$$

Solution: Divide each term in the numerator by the denominator, then use the Direct Substitution Property (DSP) and the special trigonometric limit to evaluate the limit.

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x^2 - 2x + \sin x}{x} &= \lim_{x \rightarrow 0} \left(x - 2 + \frac{\sin x}{x} \right) \\ &= \lim_{x \rightarrow 0} (x) - \lim_{x \rightarrow 0} (2) + \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \\ &= 0 - 2 + 1 \\ &= -1 \end{aligned}$$

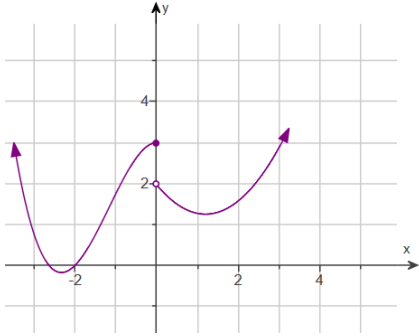
Problem 2. (3 points) Compute the limit:

$$\lim_{h \rightarrow 0^-} \left(\frac{\sqrt{6} - \sqrt{10h^2 + 9h + 6}}{h} \right)$$

Solution: Rationalize the numerator then simplify by factoring.

$$\begin{aligned} \lim_{h \rightarrow 0^-} \left(\frac{\sqrt{6} - \sqrt{10h^2 + 9h + 6}}{h} \right) &= \lim_{h \rightarrow 0^-} \left(\frac{\sqrt{6} - \sqrt{10h^2 + 9h + 6}}{h} \cdot \frac{\sqrt{6} + \sqrt{10h^2 + 9h + 6}}{\sqrt{6} + \sqrt{10h^2 + 9h + 6}} \right) \\ &= \lim_{h \rightarrow 0^-} \left(\frac{6 - (10h^2 + 9h + 6)}{h \cdot (\sqrt{6} + \sqrt{10h^2 + 9h + 6})} \right) \\ &= \lim_{h \rightarrow 0^-} \left(\frac{6 - 10h^2 - 9h - 6}{h \cdot (\sqrt{6} + \sqrt{10h^2 + 9h + 6})} \right) \\ &= \lim_{h \rightarrow 0^-} \left(\frac{h \cdot (-10h - 9)}{h \cdot (\sqrt{6} + \sqrt{10h^2 + 9h + 6})} \right) \\ &= \lim_{h \rightarrow 0^-} \left(\frac{-10h - 9}{\sqrt{6} + \sqrt{10h^2 + 9h + 6}} \right) \\ &= \frac{-9}{2 \cdot \sqrt{6}} \end{aligned}$$

Problem 3. (3 points) Use the graph of $f(x)$ below to find the following limits.



(a) $\lim_{x \rightarrow 0^-} f(x)$

(b) $\lim_{x \rightarrow 0^+} f(x)$

(c) $\lim_{x \rightarrow 0} f(x)$

Solution:

(a) $\lim_{x \rightarrow 0^-} f(x) = 3$

(b) $\lim_{x \rightarrow 0^+} f(x) = 2$

(c) $\lim_{x \rightarrow 0} f(x)$ does not exist