Quiz # 3 - Sections 2.3, 3.1 **Name:** ____________________

Math 135: Fall 2019

Instructions:
Please show all your work in order to receive proper credit. You are not allowed to use any calculator, formula sheet, notes or electronic devices. Quiz should be completed in one seating with no breaks. All final answers should be in the simplest form. Box your final answer.

Problem 1. (4 points) Find an equation of the line tangent to the graph of $y = 2x^2 - 3$ at $x = 1$; by using the limit definition of derivative.

Solution: Find the slope of the tangent line to $y = 2x^2 - 3$ at $x = 1$ by using the limit definition of the derivative. Observe that, $f(1) = 2 \cdot (1)^2 - 3 = -1$.

$$\begin{aligned}
f'(1) &= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} \\
&= \lim_{h \to 0} \frac{(2(1+h)^2 - 3) - (-1)}{h} \\
&= \lim_{h \to 0} \frac{(2 \cdot (1 + 2h + h^2) - 3) + 1}{h} \\
&= \lim_{h \to 0} \frac{(2 + 4h + 2h^2 - 2)}{h} \\
&= \lim_{h \to 0} \frac{h \cdot (4 + 2h)}{h} \\
&= \lim_{h \to 0} (4 + 2h) \\
&= 4
\end{aligned}$$

Therefore, the equation of the tangent line to $y = 2x^2 - 3$ at $x = 1$ can be found by: $y + 1 = 4(x - 1)$.

Problem 2. (3 points) For what value of *a* is the following function continuous for all x? If this is not possible, explain why.

$$f(x) = \begin{cases} -5ax & x \neq 3 \\ \sin(\frac{\pi x}{2}) & x = 3 \end{cases}$$

Solution: Notice that $f(3) = \sin(\frac{\pi \cdot 3}{2}) = -1$. We also observe that *f(x)* is defined the same to the left and to the right of the transition point, $x = 3$, therefore, left- and right-hand limits are equal. Although, for this problem it is not necessary to compute the left- and right-hand limits separately, one should state such an observation before directly computing the two-sided limit. Therefore,we have:

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (-5ax) = -5a \cdot 3 = -15a$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (-5ax) = -5a \cdot 3 = -15a$$

We observe that the one-sided limits are equal. In order for $f(x)$ to be continuous for all x, $f(x)$ must be continuous at the transition point of $x = 3$.

Therefore, the one-sided limits and $f(3)$ must be equal. So, we find that: $-15a = -1$, or $a = \dfrac{1}{15}$.

Problem 3. (3 points) Use the limit definition of derivative to calculate the derivative of $f(x) = \dfrac{2}{x-3}$ at $x = 5$. Simplify your answer.

Solution: Observe that $f(5) = 1$.

$$
\begin{aligned}
f'(5) &= \lim_{x \to 5} \frac{f(x) - f(5)}{x - 5} \\
&= \lim_{x \to 5} \frac{\frac{2}{x-3} - 1}{x - 5} \\
&= \lim_{x \to 5} \frac{\frac{2-(x-3)}{x-3}}{x - 5} \\
&= \lim_{x \to 5} \frac{\frac{5-x}{x-3}}{x - 5} \\
&= \lim_{x \to 5} \frac{-1}{x - 3} \\
&= \frac{-1}{2}
\end{aligned}
$$

Then, the derivative of $f(x) = \dfrac{2}{x-3}$ at $x = 5$ is $-\dfrac{1}{2}$.