

Supplementary Problem

Q19 (Level B) on 4.3.

Sketch the graph of $f(x) = 1 + 2x + 18x^{-1}$ by determining where the function is decreasing or increasing; where it's concave up/down.

$$f(x) = 1 + 2x + 18x^{-1} = 1 + 2x + \frac{18}{x} = \frac{x + 2x^2 + 18}{x}$$

$$f'(x) = 2 - 18x^{-2}$$

V.A: $x=0$

H.A: none ($\frac{2x^2}{0x^2}$)

First-order critical #s: $f'(x) = 0$ or DNE

$$f'(x) = 0 = 2 - \frac{18}{x^2} \Rightarrow 2 = \frac{18}{x^2} \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$$

$f'(x)$ DNE $\Rightarrow x=0$ (however, function is NOT defined & continuous at $x=0$)

Sign chart for $f'(x)$

$$f'(x) = 2 - \frac{18}{x^2}$$

	-3	0	3		
Sign of $f'(x)$	+	\emptyset	-	\emptyset	+
incr/dec.	↘		↗		

Local max at $x = -3$; Local min at $x = 3$

$f(x)$ is increasing on $(-\infty, -3)$, $(3, \infty)$

$f(x)$ is decreasing on $(-3, 0)$, $(0, 3)$

$$f'(x) = 2 - 18x^{-2}$$

$$f''(x) = 36x^{-3}$$

Second-order critical #s: $f''(x) = 0$ or DNE
none *none*

$$f'''(x) = \frac{36}{x^3}$$

(f is not defined at $x=0$)

When constructing sign chart for $f''(x)$
 (include x -coordinates of V.A. also)

$$f''(x)$$

	<u>0</u>
sign of $f''(x)$	- +
concave up/down	∩ ∪

Since $x=0$ is NOT defined on $f(x)$

$x=0$ is NOT a PoI.

$f(x)$ is concave down on $(-\infty, 0)$

$f(x)$ is concave up on $(0, \infty)$

Why H.A. is *none*?

$$\lim_{x \rightarrow \infty}$$

$$\frac{1 + 2x + 18}{1} \rightarrow \infty$$

$$\lim_{x \rightarrow -\infty}$$

$$\frac{1 + 2x + 18}{1} \rightarrow -\infty$$

since $\infty \neq -\infty$

H.A. \rightarrow none

$$f(x) = 1 + 2x + 18x^{-1}$$

Put everything together to graph $f(x)$:

local min Point: $(3, f(3)) \Rightarrow (3, 13)$

$$f(3) = 1 + 6 + 18 \cdot 3^{-1} = 7 + 6 = 13$$

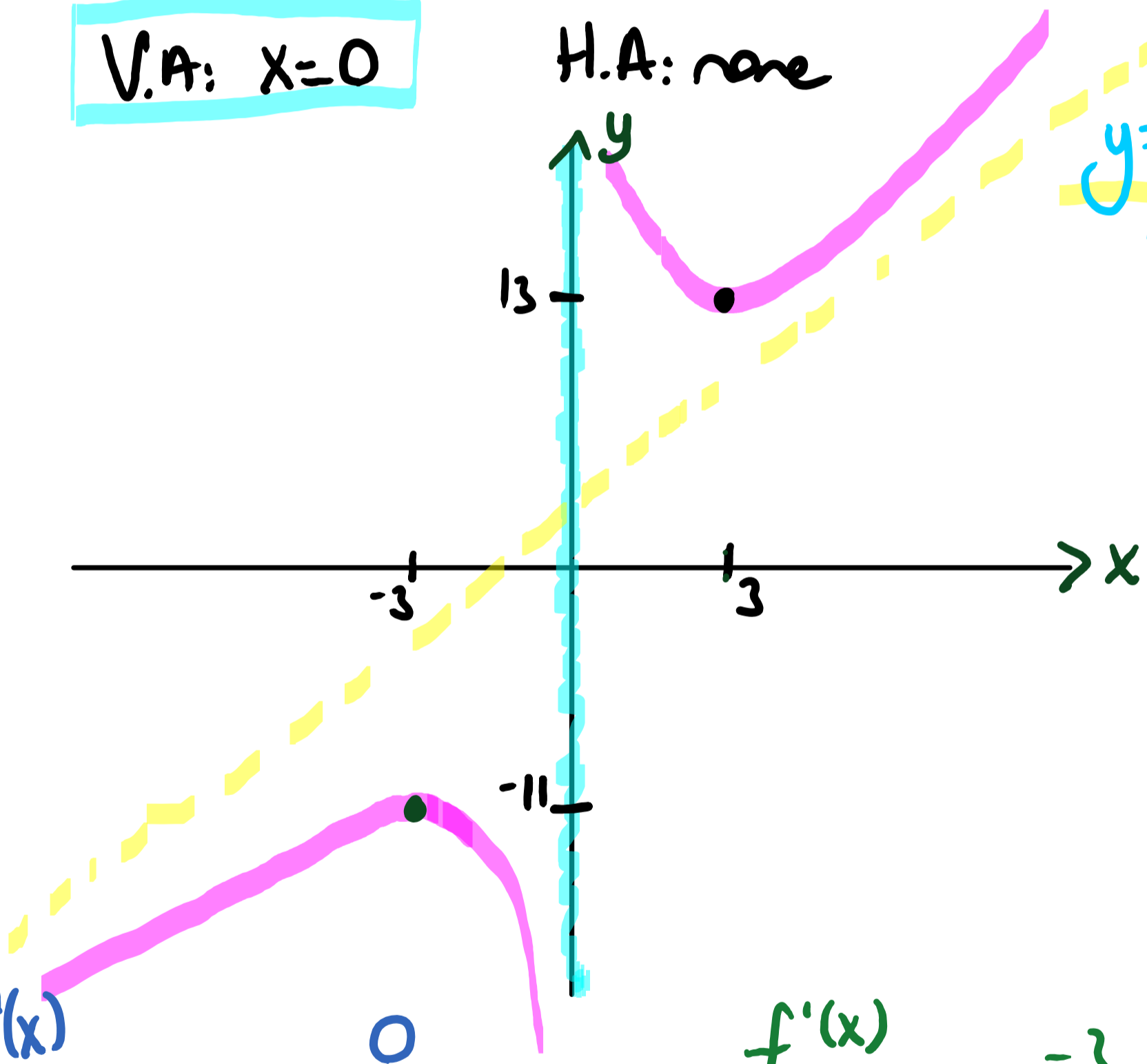
local max Point: $(-3, f(-3)) \quad (-3, -11)$

$$f(-3) = 1 - 6 + 18(-3)^{-1} = -5 - 6 = -11$$

V.A: $x=0$

H.A: none

$y = 2x + 1$
Slant Asymptote



$f'''(x)$	0	
sign of $f'''(x)$	-	+
concave up/down	∩	∪

$f'(x)$	-3	0	3
sign of $f'(x)$	+	0	-
inc/dec.	↗	↘	↗