# MIDTERM REVIEW

## 4.1. Extreme Values

Exp) Find the absolute maximum value and the abs. minimum value of

$$g(x) = x^{2/3} \cdot (5-2x) \quad \text{on } [-1, 2]$$

Step 1) $f'(x)$ and find critical #s

$$f'(x) = (x^{2/3})'(5-2x) + (x^{2/3})(5-2x)'$$

$$= \frac{2}{3} \cdot x^{-1/3}(5-2x) + x^{2/3}(-2)$$

$$= 2 \cdot x^{-1/3}\left[\frac{1}{3}(5-2x) + x^{1} \cdot (-1)\right]$$

$x^4 + x^3 = x^3(x+1)$

$$= 2x^{-1/3}\left(\frac{5}{3} - \frac{2x}{3} - \frac{x}{1}\frac{(3)}{(3)}\right)$$

$x^{-1/3} = \frac{1}{x^{1/3}}$

$$= 2x^{-1/3}\left(\frac{5}{3} - \frac{2x}{3} - \frac{3x}{3}\right)$$

$x^{1/3} = \sqrt[3]{x}$

$$= 2x^{-1/3}\left(\frac{5}{3} - \frac{5x}{3}\right) = \frac{2}{\sqrt[3]{x}}\left(\frac{5-5x}{3}\right)$$

$$f'(x)=\frac{2}{\sqrt[3]{x}}\left(\frac{5-5x}{3}\right)=\frac{2}{\sqrt[3]{x}}\cdot\frac{5(1-x)}{3}=\frac{10(1-x)}{3\cdot\sqrt[3]{x}}$$

Critical #s: $f'(x)=0$

$$\frac{10(1-x)}{3\sqrt[3]{x}}=\frac{0}{1}$$

$$10(1-x)=0$$

$$\boxed{x=1}$$

Critical #

$f'(x)$ DNE

$3\sqrt[3]{x} \rightarrow$ DNE

critical #

when $\boxed{x=0}$

$$f'(x)=\frac{10(1-x)}{\underbrace{3\sqrt[3]{x}}_{\text{DNE}}}$$

Step2: find y-values for critical #s, endpoints

$[-1,2]$

| x | f(x) |
|---|---|
| 1 | $1^{2/3}\cdot(5-2\cdot 1)=3$ |
| 0 | 0 |
| -1 | $(-1)^{2/3}(5+2\cdot(+1))=\boxed{7}$ |
| 2 | $2^{2/3}(5-2\cdot 2)=2^{2/3}$ |

$$f(x)=x^{2/3}(5-2x)$$

$$(-1)^{2/3}=\sqrt[3]{(-1)^2}=1$$

The abs. max of $f(x)$ is 7.

The abs. min of $f(x)$ is 0.

Precalc

Solve the inequality in interval notation

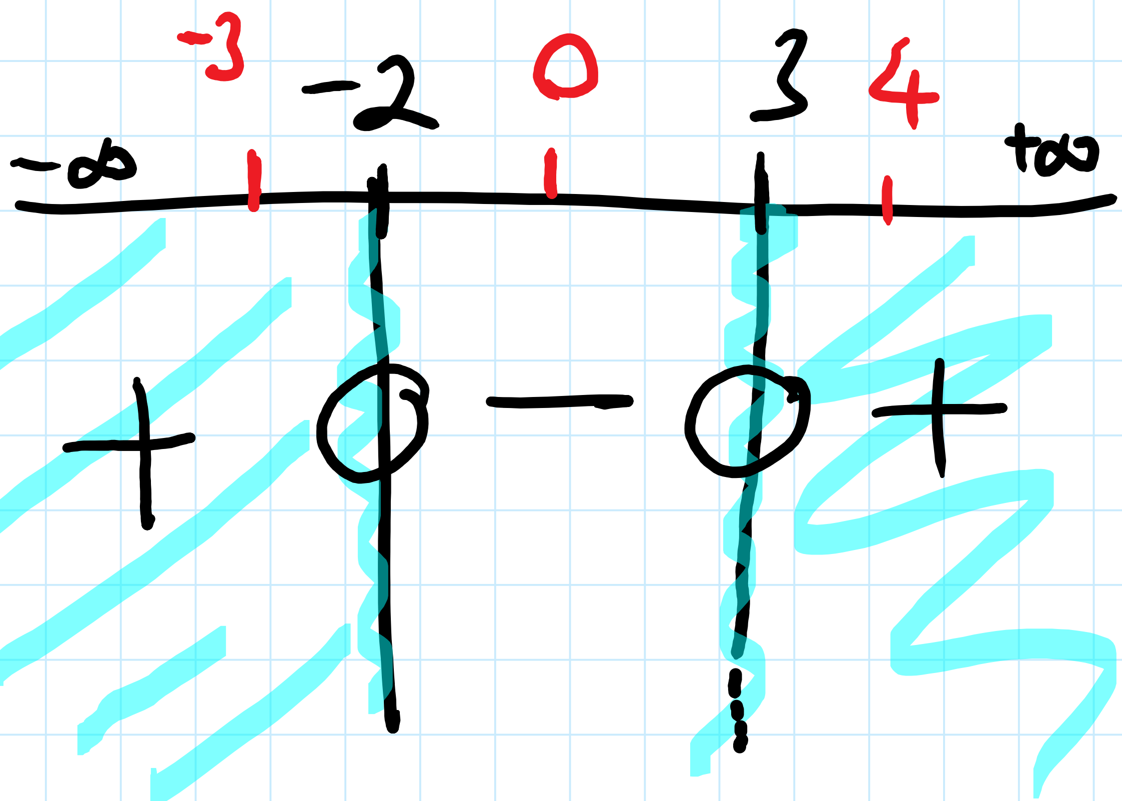
$x^2 - x \geq 6$

Solution:

$x^2 - x - 6 \geq 0$

$-3 \quad 2$

$(x-3)(x+2) \geq 0$




cutpoints: $x = 3, -2$

testpoints: $x = -3, 0, 4$

$f(-3) = -6 \cdot -1 = 6 > 0$

$f(0) = -3 \cdot 2 = -6$

$(-\infty, -2] \cup [3, \infty)$

Exp) Solve for x:

1st method: re-write as expo.

$\log_2 2^{x^2} = 4 \Rightarrow 2^4 = 2^{x^2}$

$4 = x^2$

$\boxed{x = \mp 2}$

2nd method: use prop. of log.

$\log_2 2^{\boxed{x^2}} = x^2 \cdot \cancel{\log_2 2}^{\,1} = 4 \Rightarrow x^2 = 4$

## Continuity

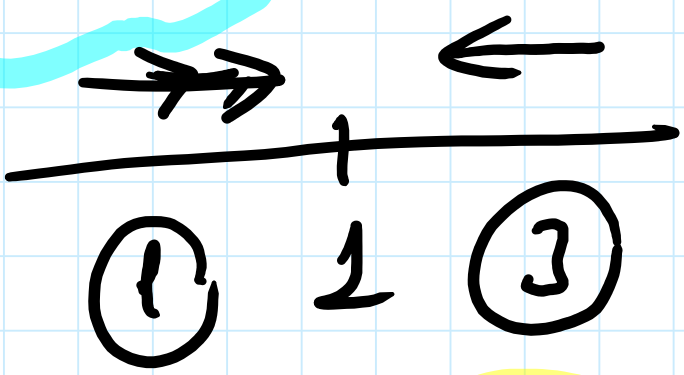
Find constants $A, B$ such that $f$ is continuous for all $x$:

$$f(x) = \begin{cases} Ax+3, & \text{if } x<1 \quad (1) \\ 5, & \text{if } x=1 \quad (2) \\ x^2+B, & x>1 \quad (3) \end{cases}$$

check the transition points

$$\boxed{\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)}$$

$$\lim_{x\to 1} f(x)$$




$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (Ax+3) = A\cdot(1)+3 = A+3$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x^2+B) = 1^2+B = 1+B$$

$$f(1) = 5$$

$$A+3 = 5 = 1+B$$

$$\boxed{A=2 \qquad B=4}$$

# Limits

Exp) $\lim_{x \to e} \frac{\ln\sqrt{x}}{x}$

"D.S.P"

$$\frac{\ln\sqrt{e}}{e} = \frac{\ln(e)^{1/2}}{e}$$

$$= \frac{\frac{1}{2} \cdot \ln e}{e}$$

$$= \frac{\frac{1}{2} \cdot 1}{e} = \frac{1}{2e}$$

$1$

Exp) $\lim_{x \to 0} \frac{\sin(3x)}{\sin(2x)}$

$$= \lim_{x \to 0} \frac{\sin(3x)}{3x} \cdot 3x \cdot \frac{1}{\sin(2x)} 2x \cdot \frac{1}{2x}$$

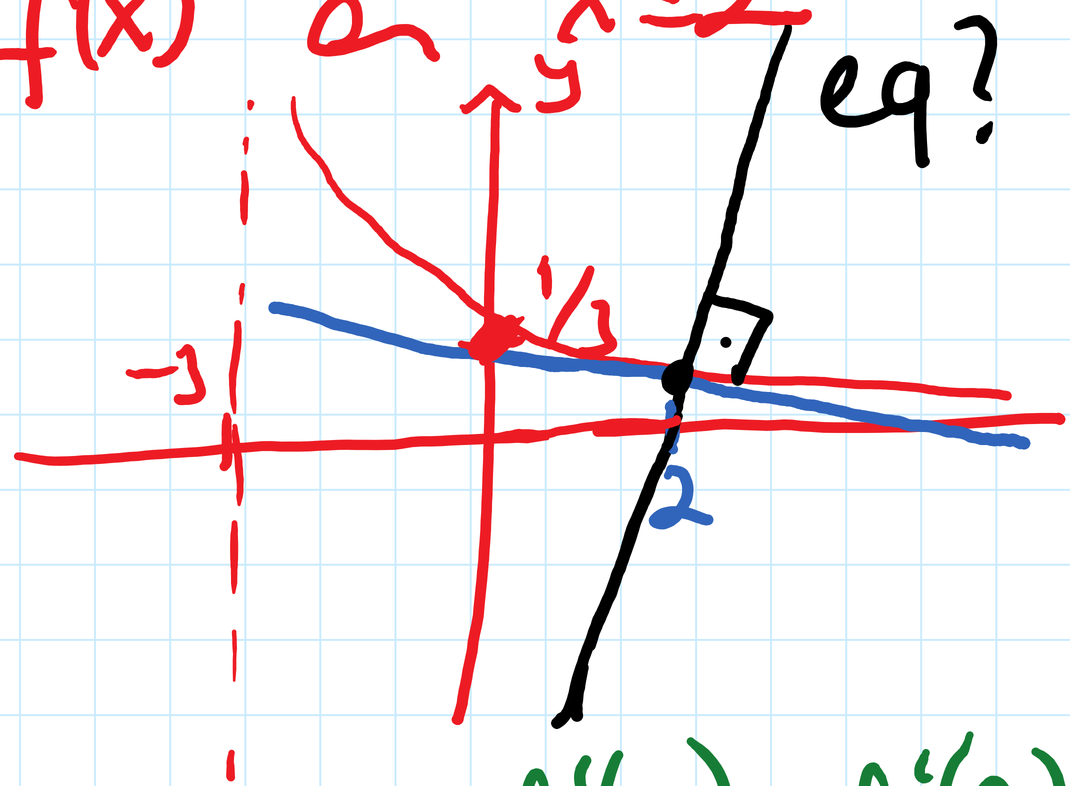
$$= \lim_{x \to 0} \frac{3x}{2x} = \lim_{x \to 0} \frac{3}{2} = \frac{3}{2}$$

## Recall: Special Trig Limits

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \quad ; \quad \lim_{x \to 0} \frac{x}{\sin x} = 1$$

Exp) Find an equation of the normal line to $f(x)$ on $x=2$ by using limit def. of der.

$f(x) = \frac{1}{x+3}$




$m_{tan}\Big|_{x=2} = ?$

$m_{normal} = \frac{-1}{m_{tan}}$

eq → $y - y_1 = m(x - x_1)$

$f'(x) \to f'(2)$

$f(x) = \frac{1}{x+3}$

$f(2) = \frac{1}{5}$

$$f'(2) = \lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$$

$$f'(2) = \lim_{x\to 2} \frac{\frac{1}{x+3}(5) - \frac{1}{5}(x+3)}{x-2} = \lim_{x\to 2} \frac{\frac{5-(x+3)}{5(x+3)}}{\frac{x-2}{1}}$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{x\to 2} \frac{5-(x+3)}{5(x+3)} \cdot \frac{1}{x-2}$$

$$2-x=-(x-2)$$

$$f'(2)=\lim_{x\to 2}\frac{5-x-3}{5(x+3)(x-2)}=\lim_{x\to 2}\frac{2-x}{5(x+3)(x-2)}$$

$$=\lim_{x\to 2}\frac{-\cancel{(x-2)}}{5(x+3)\cancel{(x-2)}}=\lim_{x\to 2}\frac{-1}{5(x+3)}$$

"DSP"

$$m_{tan}\Big|_{x=2}=\frac{-1}{5(2+3)}=\frac{-1}{25}$$

$$m_{normal}=\frac{-1}{m_{tan}}\Bigg|_{x=2}$$

$$m_{normal}\Big|_{x=2}=\frac{-1}{\frac{-1}{25}}=25$$

$x=2$ $\quad f(2)=\frac{1}{2+3}=\frac{1}{5}$ $\quad m_{normal}=25$

$f(x)=\frac{1}{x+3}$ $\quad \left(2,\frac{1}{5}\right)$

$$y-y_1=m_{normal}(x-x_1)$$

$$y-\frac{1}{5}=25(x-2)$$

Exp) Find the equation for the tangent line to the curve with equation $y = x^4 - 2x + 1$ that's parallel to the line $2x - y - 3 = 0$

Find eq. of $2x - y - 3 = 0$

$2x - 3 = y \quad m = 2$

If 2 lines are // then $m_1 = m_2$

$m = 2 \Rightarrow m = 2$

$m_{tan}\Big|_{x?} = 2 = (x^4 - 2x + 1)'$

$2 = 4x^3 - 2$

$\frac{4}{4} = \frac{4x^3}{4} \Rightarrow x^3 = 1 \Rightarrow \boxed{x = 1}$

$f(1) = 1^4 - 2 \cdot 1 + 1 = 0$

$(1, 0)$
$(x_1, y_1)$
$m = 2$

$y - y_1 = m(x - x_1)$

$y - 0 = 2(x - 1)$

Find the derivative.
After finding the derivative, don't simplify.

Exp) $g(x) = \frac{1}{2\sqrt{x}} + \frac{x^2}{4} + \pi^5$

$g'(x) = \left(\frac{1}{2}\cdot x^{-1/2} + \frac{x^2}{4} + \pi^5\right)'$

$\to \quad = \frac{1}{2}\cdot\frac{-1}{2}\cdot x^{-3/2} + \frac{1}{4}\cdot 2x + 0)$

$= \frac{-1}{4}\cdot x^{-3/2} + \frac{x}{2}$

Exp) $f(x) = \ln(\sin x + \cos x)$

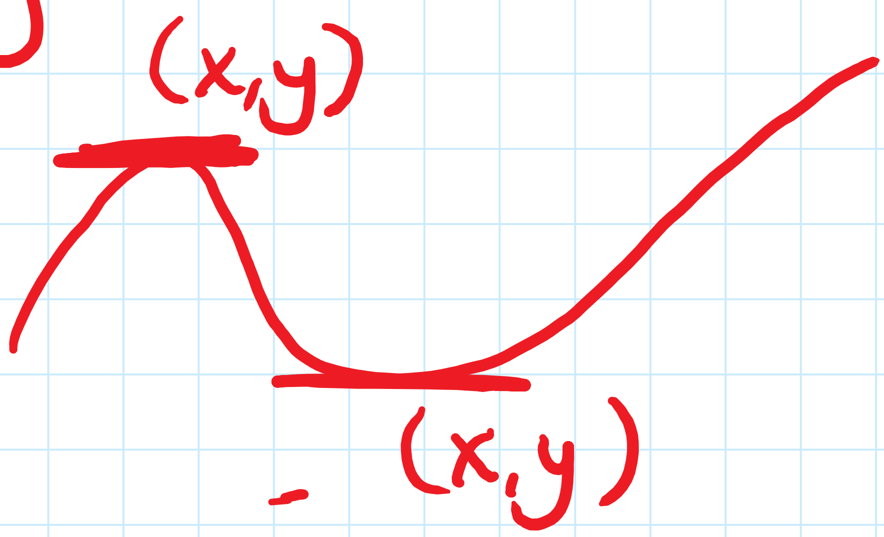
$\ln(f(x))' = \frac{f'(x)}{f(x)}$

$f'(x) = \frac{(\sin x + \cos x)'}{\sin x + \cos x}$

$f'(x) = \frac{\cos x - \sin x}{\sin x + \cos x}$

Exp) Find the coordinates $(x,y)$ of each point on the graph of $h(u)$ where the tangent line is horizontal.

$$h(u) = \frac{1}{\sqrt{u}} \cdot (u+9)$$




Horizontal tangent: $h'(u) = 0$

$$h'(u) = \left[u^{-1/2} \cdot (u+9)\right]'$$

$$= \left[u^{1/2} + 9u^{-1/2}\right]'$$

$$= \frac{1}{2} \cdot u^{-1/2} + 9 \cdot \frac{-1}{2} \cdot u^{-3/2}$$

$$= \frac{1}{2} u^{-3/2} \left[u^{1} - 9\right] = 0$$

$$= \frac{1}{2\sqrt[2]{u^3}} \qquad [u-9] = 0$$

$$u - 9 = 0 \Rightarrow \boxed{u = 9}$$

$x^2 + x^3 = x^2(x+1)$

$$h(u) = \frac{1}{\sqrt{u}}(u+9)$$

$$h(9) = \frac{1}{\sqrt{9}}(9+9) = \frac{1}{3} \cdot 18 = 6$$

$$(u, h(u)) = (9, 6)$$

Let $g(x) = 6 - \frac{9}{x}$, calculate $g'(3)$ by using limit def. of derivative.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \qquad f(x+h) = 6 - \frac{9}{x+h}$$

$$\lim_{h\to 0} \frac{6 - \frac{9}{x+h} - \left(6 - \frac{9}{x}\right)}{h}$$

$$\lim_{h\to 0} \frac{\cancel{6} - \frac{9}{x+h} - \cancel{6} + \frac{9}{x}}{h} = \lim_{h\to 0} \frac{\frac{-9}{x+h}_{\times x} + \frac{9}{x}_{\times (x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{-9x}{x(x+h)} + \frac{9(x+h)}{x(x+h)}}{h} = \lim_{h\to 0} \frac{\frac{\cancel{-9x} + \cancel{9x} + 9h}{x(x+h)}}{\frac{h}{1}}$$

$$= \lim_{h\to 0} \frac{9\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}} = \lim_{h\to 0} \frac{9}{x(x+h)} = \frac{9}{x^2}$$

$$\left.\frac{9}{x^2}\right|_{x=3} = \frac{9}{3^2} = 1$$

$$g'(3) = 1$$