Math 135 - Quiz #4, Sections 3.2, 3.3, 3.5

Solutions:

Instructions:
Show all your work in order to receive proper credit. No formula sheets are allowed during any quizzes. No cell phones, calculators, or any other electronic devices are allowed in a student's possession during any quiz. All such devices must be put away in the student's bag, out of reach of the student during the quiz, with cell phones turned completely off (not just on vibrate or silent). If a student's cell phone rings during a quiz, or if a student is in possession of an unauthorized electronic device during a quiz, they may have to hand in the quiz and not be allowed to complete it, and a report of an academic integrity violation may be filed. Quiz should be completed in one seating with no breaks. Box your final answer. Good luck!
Timing:10 minutes

Problem 1. (3 points) Find the second derivative of the function $f(x) = x^3 \cdot e^{5x}$. *Do not simplify your answer.*

Solution: Find the first derivative of $f(x)$ by using the product rule with a chain rule for e^{5x}.
$f'(x) = 3x^2e^{5x} + 5x^3e^{5x}$
Find the second derivative of $f(x)$ by using the product rule again for each term of the sum with a chain rule for e^{5x}.
$f''(x) = 6xe^{5x} + 15x^2e^{5x} + 15x^2e^{5x} + 25x^3e^{5x}$

Problem 2. (3 points) Find the derivative of the function $g(t) = \dfrac{ln(2t)}{\sqrt{t}}$. *Do not simplify your answer.*

Solution: Find the derivative of $g(t)$ by using the quotient rule, the chain rule for the numerator and the power rules for the denominator. Remember to rewrite the radical function $\sqrt{t}$ as a power function, $t^{\frac{1}{2}}$, to differentiate it easier. So, $g(t) = \dfrac{ln(2t)}{t^{\frac{1}{2}}}$.

$g'(t) = \dfrac{\frac{2}{2t} \cdot t^{\frac{1}{2}} - ln(2t) \cdot \frac{1}{2} \cdot t^{-\frac{1}{2}}}{t}$ or $g'(t) = \dfrac{\frac{t^{\frac{1}{2}}}{t} - \frac{ln(2t) \cdot t^{-\frac{1}{2}}}{2}}{t}$

Problem 3. (4 points) Find the equation of the tangent line to $h(x) = \sqrt{x} - \dfrac{1}{x}$ at $x = 4$. *Any form of the equation of a line is acceptable.*

Solution: Observe that $h(4) = \sqrt{4} - \dfrac{1}{4} = 2 - \dfrac{1}{4} = \dfrac{7}{4}$.

Therefore, the point of tangency is $\left(4, \frac{7}{4}\right)$. The slope of the tangent line at $(4, \frac{7}{4})$ is $h'(4)$.
Before taking the derivative of $h(x)$, it would be a good strategy to re-write $h(x)$ as a power function. Therefore, $h(x) = x^{\frac{1}{2}} - x^{-1}$. Compute the derivative of $h(x)$ by using the power rule only.

$$h'(x) = \frac{1}{2}x^{-\frac{1}{2}} - (-1) \cdot x^{-2} = \frac{1}{2}x^{-\frac{1}{2}} + x^{-2}$$

$$h'(4) = \frac{1}{2} \cdot 4^{-\frac{1}{2}} + 4^{-2} = \frac{1}{2} \cdot \frac{1}{4^{\frac{1}{2}}} + \frac{1}{4^2} = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{4^2} = \frac{1}{4} + \frac{1}{16} = \frac{5}{16}$$

Then, the equation of the tangent line to $h(x)$ at $x = 4$ is:

$$y - \frac{7}{4} = \frac{5}{16}(x - 4).$$