Math 135, Quiz #5 Solutions

Name: ______________________ Section: ______________________

Instructions: Show all your work in order to receive proper credit. No formula sheets and no notes are allowed during the quiz. No cell phones, calculators, or any other electronic devices are allowed in a student's possession during any quiz. All such devices must be put away in the student's bag, out of reach of the student during the quiz. Quiz should be completed in one seating with no breaks. Box your final answer. Good luck!
Timing: 15 minutes

1. **(3 pts)** Use implicit differentiation to find y'.

$$y \cdot \sqrt{x+1} = 4$$

Solution: Differentiate implicitly with respect to x to obtain:

$$y \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x+1}} + y' \cdot \sqrt{x+1} = 0$$

Solve for y':

$$y' = -\frac{y \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x+1}}}{\sqrt{x+1}}$$

By simplifying y', we obtain:

$$y' = -\frac{y}{2(x+1)}$$

2. **(3 pts)** Find the slope of the tangent line to the graph of $5x^2y - \pi \cdot \cos(y) = 6\pi$ at $P(1, \pi)$.

Solution: Differentiate implicitly with respect to x to obtain:

$$10xy + 5x^2y' + \pi \sin(y)y' = 0$$

Solve for y':

$$y' \cdot (5x^2 + \pi \sin(y)) = -10xy$$

$$y' = \frac{-10xy}{5x^2 + \pi \sin(y)}$$

By substituting $P(1, \pi)$ in y', we obtain:

$$y' = \frac{-10 \cdot 1 \cdot \pi}{5 \cdot 1^2 + \pi \sin(\pi)} = -2\pi$$

At $P(1, \pi)$, the slope of the tangent line to the graph is -2π.

3. **(4 pts)** Find the absolute minimum value and absolute maximum value of $g(x) = \dfrac{5x}{x^2+1}$ on $[-2, 1]$.

Solution: Since $g(x)$ is continuous and differentiable on $[2, 1]$, the only critical numbers of $g(x)$ are solutions to $g'(x) = 0$.
We find $g'(x)$ by using the quotient rule:

$$g'(x) = \frac{5(x^2+1) - 5x \cdot 2x}{(x^2+1)^2} = \frac{-5x^2+5}{(x^2+1)^2} = \frac{-5 \cdot (x^2-1)}{(x^2+1)^2}$$

The critical numbers of $g(x)$ are: $x = 1, x = -1$ which are both in $[-2, 1]$. Now we compare the endpoint values and the critical values: $g(-2) = -2$, $g(-1) = -\frac{5}{2}$, and $g(1) = \frac{5}{2}$.
The absolute minimum value of $g(x)$ on $[2, 1]$ is $-\frac{5}{2}$ and the absolute maximum value of $g(x)$ on $[2, 1]$ is $\frac{5}{2}$.