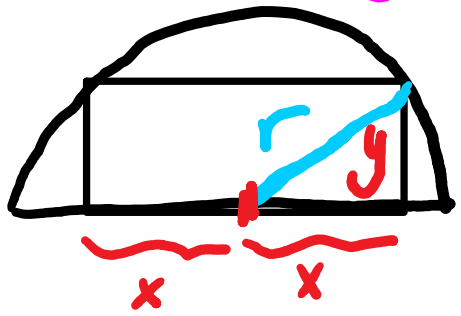
Exp) Q15 on Page 291

Find the rectangle of largest area that can be inscribed in a semicircle of radius 10cm, assuming that one side of the rectangle lies on the diameter of the semicircle.

Draw a figure



r=10cm.

Objective function & Constraints

Area of a rectangle is length x width.

In our example, dimensions of the rectangle are:

2x by y.

Objective function: $A(x,y) = 2x \cdot y$

Constraint: $x^2+y^2=r^2 \Rightarrow x^2=r^2-y^2$

$$x=\sqrt{r^2-y^2}$$

Objective function: $A(y) = 2\cdot\sqrt{r^2-y^2}\cdot y$

(recall $r=10$ cm)

$$A(y)=2\sqrt{100-y^2}\cdot y$$

The objective is to find the y-value that gives the MAXIMUM area.

$A'(y) = 0$ or DNE

$A(y) = 2y\sqrt{100-y^2}$ (use product rule)

$A'(y) = 2\sqrt{100-y^2} + 2y \cdot -2y \cdot \frac{1}{2}(100-y^2)^{-1/2}$

$= 2\sqrt{100-y^2} - 2y^2(100-y^2)^{-1/2}$

$= \frac{2\sqrt{100-y^2}}{1} - \frac{2y^2}{\sqrt{100-y^2}}$

$\left(\sqrt{100-y^2}\right)$

$= \frac{2(100-y^2) - 2y^2}{\sqrt{100-y^2}} = \frac{200-4y^2}{\sqrt{100-y^2}}$

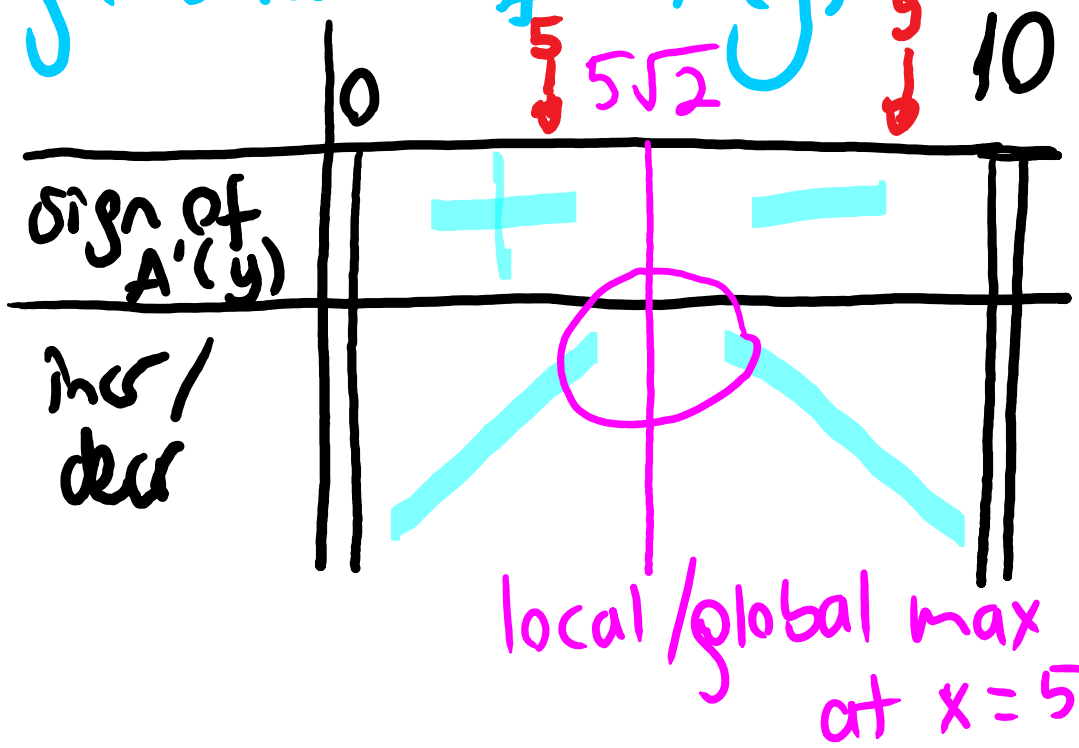
$A'(y) = 0 \Rightarrow 200-4y^2 = 0 \Rightarrow y = \sqrt{50} = 5\sqrt{2}$

$A'(y)$ DNE $\Rightarrow \sqrt{100-y^2} = 0 \Rightarrow y = 10$

first-order critical #

Domain of y: $[0, 10]$

sign chart for $A'(y)$



test points: $x = 5, 9$

$A'(y) = \frac{200-4y^2}{\sqrt{100-y^2}}$

$A'(5) = \frac{\oplus}{\oplus} = \oplus$

$A'(9) = \frac{\ominus}{\oplus} = \ominus$

$y = 5\sqrt{2}$ is actually the local/global max of $A(y)$.

$$A(y) = 2y\sqrt{100 - y^2}$$

$$A(5\sqrt{2}) = 2 \cdot 5\sqrt{2}\left(\sqrt{100 - (5\sqrt{2})^2}\right)$$

$$= 10\sqrt{2} \cdot \sqrt{100 - 25 \cdot 2}$$

$$= 10\sqrt{2} \cdot \sqrt{50}$$

$$= 10 \cdot \sqrt{100} = 100 \text{ cm}^2 \text{ is the largest area.}$$

Dimensions of the rectangle with the largest area is: $2x$ by y ($10\sqrt{2}$ cm by $5\sqrt{2}$ cm)

$$2x = 2 \cdot \sqrt{100 - y^2} \Rightarrow 2x = 2\sqrt{100 - (5\sqrt{2})^2}$$

$$y = 5\sqrt{2}$$

$$= 2\sqrt{100 - 25 \cdot 2}$$

$$= 2\sqrt{50} = 2 \cdot 5\sqrt{2} = 10\sqrt{2}$$