Math 135, Quiz #6 Solutions

Name: ________________________ Section: ________________________

Instructions: Show all your work in order to receive proper credit. No formula sheets and no notes are allowed during the quiz. No cell phones, calculators, or any other electronic devices are allowed in a student's possession during any quiz. All such devices must be put away in the student's bag, out of reach of the student during the quiz. Quiz should be completed in one seating with no breaks. Box your final answer. Good luck!
Timing: 15 minutes

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1. **(3 pts)** Use linear approximation to estimate $(1.005)^{50}$.

Solution:
Let $f(x) = x^{50}$. Our goal is to estimate $f(1.005)$ by using the linearization of the function at a point $x = a$ which is $L(x) = f(a) + f'(a) \cdot (x - a)$. Here, $f(a)$ represents the value of the function $f(x)$ which we can compute without using a calculator that is also very close to the unknown value. In other words: $a = 1$, $x = 1.005$.

$$f(x) = x^{50}$$
$$f(1) = 1^{50} = 1$$
$$f'(x) = 50 \cdot x^{49}$$
$$f'(1) = 50 \cdot 1^{49} = 50$$

Therefore, $L(x)$ function can be re-written as:

$$\begin{aligned} L(x) &= f(1) + f'(1) \cdot (x - 1) \\ &= 1 + 50(x - 1) \end{aligned}$$

Our last step is to approximate $L(1.005)$ by simply substituting 1.005 into $L(x)$. Therefore,

$$(1.005)^{50} = f(1.005) \approx 1 + 50(1.005 - 1) = 1.25$$

2. **(3 pts)** Water is falling on a surface, wetting a circular area that is expanding at a rate of 18π mm^2/s. How fast is the radius of the wetted area expanding when the radius is 12 mm? You must include correct units as part of your answer.

Solution: Let r denote the radius of the wetted area. Since the wetted area has a circular shape, its area is given by

$$A = \pi r^2$$

Differentiating both sides implicitly with respect to time t yields:

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

Our goal is to find $\frac{dr}{dt}$ when $r = 12$ mm and $\frac{dA}{dt} = 18\pi$ mm^2/s. We substitute the specific situation values:

$$18\pi = 2\pi \cdot 12 \cdot \frac{dr}{dt}$$

Solving for $\frac{dr}{dt}$ provides:

$$\frac{dr}{dt} = \frac{3}{4}$$

This means that the radius of the wetted area is expanding at a rate of 0.75 mm/s.

3. **(4 pts)** A manufacturer's total cost is:

$$C(q) = 3q^2 + q + 500$$

dollars, where q is the number of units produced.
a. Use marginal analysis to *estimate* the cost of manufacturing the 41st unit. You must include correct units as part of your answer.
b. Compute the *actual cost* of manufacturing the 41st unit. You must include correct units as part of your answer.

Solution:

a. The cost of manufacturing the 41st unit is the change in cost as q increases from 40 to 41 and is estimated by $MC(40) \approx C'(40)$. Therefore, first we find $C'(q) = 6q + 1$, then we substitute 40 for q which produces $MC(40) \approx C'(40) = 6 \cdot 40 + 1 = 241$.
Therefore, we estimate the cost of manufacturing the 41st unit to be \$241.

b. The actual cost is:

$$\begin{aligned} \Delta C &= C(41) - (40) \\ &= (3 \cdot (41)^2 + 41 + 500) - (3 \cdot (40)^2 + 40 + 500) \\ &= 3(41^2 - 40^2) + (41 - 40) + (500 - 500) \\ &= 3(41 - 40)(41 + 40) + 1 \\ &= 244 \end{aligned}$$

Therefore, we compute the actual cost of manufacturing the 41st unit as \$244.