Math 135, Quiz #8 Solutions

Name: ____________________ Section: ____________________

Instructions: Show all your work in order to receive proper credit. No formula sheets and no notes are allowed during the quiz. No cell phones, calculators, or any other electronic devices are allowed in a student's possession during any quiz. All such devices must be put away in the student's bag, out of reach of the student during the quiz. Quiz should be completed in one seating with no breaks. Box your final answer. Good luck!
Timing: 15 minutes

1. **(6 pts)** You must use calculus methods to solve the problem, and you must justify that your answer really does give the smallest sum.
   The product of two positive numbers is 36. Find the smallest value of their sum.

Solution:

Let x and y be the positive numbers. Our goal is to minimize the function $S(x, y) = x+y$. In other words, the objective function is $S(x, y) = x + y$ and the constraint is $x \cdot y = 36$. Next, we use the constraint to re-write one variable into another such as $x = \frac{36}{y}$. Therefore, the objective function can now be re-written in terms of a single variable, x as: $S(y) = \dfrac{36}{y} + y$. The domain of y is $(0, \infty)$ since there are no maximum values stated for x or y, although the problem states that both numbers are positive numbers. Our next step is to find the first-order critical numbers of $S(y)$ on $(0, \infty)$. Since $S(y)$ is differentiable on its domain, the only critical numbers of $S(y)$ are the solutions to $S'(y) = 0$.

$$S'(y) = 1 - \frac{36}{y^2} = 0$$

$$\frac{y^2 - 36}{y^2} = 0$$

$$y^2 - 36 = 0$$

$$y = 6$$

We do not consider $y = -6$ as a solution since y is a positive number.

In order to ensure the sum of x and y produces the smallest value when $y = 6$, we have to check the validity of our claim by using calculus such as:
(1) construct a sign chart for the first-derivative of the objective function
(2) investigate the concavity of the second-derivative of the objective function
(3) refer to the end points and the first-order critical numbers of the objective function to compare the values. In this problem, it is easier to find $S''(y)$ for concavity test.

$$S''(y) = \frac{72}{y^3}$$

since $y > 0$, $S''(y)$ is always a positive number. Therefore, the graph of $S(y)$ is concave up on the domain of the function $S(y)$. This implies that $y = 6$ actually produces the global minimum of $S(y)$. In conclusion, the smallest sum is:

$$S(6) = \frac{36}{6} + 6 = 12$$

2. Evaluate the limit or determine that it does not exist. If the limit is infinite, then your answer should be ∞ or -∞ (as appropriate), instead of does not exist. If you use the L'Hôpital's Rule, justify the use of it. Each part is 2 points.

(Part a) $\lim_{x\to 0}\left(\frac{x\tan(x)}{\sin(3x)}\right)$

Solution: $\lim_{x\to 0}\left(\frac{x\tan(x)}{\sin(3x)}\right)$

When we use "DSP - direct substitution property", we obtain $\frac{0}{0}$. Therefore, we can use L'Hôpital's Rule.

$$\lim_{x\to 0}\left(\frac{x\tan(x)}{\sin(3x)}\right) = \lim_{x\to 0}\left(\frac{\tan(x)+x\cdot(\sec(x))^2}{3\cos(3x)}\right)$$

When we use DSP, we obtain

$$\lim_{x\to 0}\left(\frac{\tan(x)+x\cdot(\sec(x))^2}{3\cos(3x)}\right) = \left(\frac{0+0}{3\cdot 1}\right) = 0$$

Therefore, $\lim_{x\to 0}\left(\frac{x\tan(x)}{\sin(3x)}\right) = 0$

(Part b) $\lim_{x\to\infty}\left(\frac{3+\ln(x)}{x^2+7}\right)$

Solution: When we use "DSP - direct substitution property", we obtain $\frac{3+\infty}{\infty+7} = \frac{\infty}{\infty}$. Therefore, we can use L'Hôpital's Rule.

$$\lim_{x\to\infty}\left(\frac{3+\ln(x)}{x^2+7}\right) = \lim_{x\to\infty}\left(\frac{\frac{1}{x}}{2x}\right) = \lim_{x\to\infty}\left(\frac{1}{2x^2}\right)$$

When we use DSP, we obtain

$$\lim_{x\to\infty}\left(\frac{1}{2x^2}\right) = \left(\frac{1}{\infty}\right) = 0$$

Therefore, $\lim_{x\to\infty}\left(\frac{3+\ln(x)}{x^2+7}\right) = 0$