**Math 135, Quiz #9 Solutions**

Name: ______________________ Section: ______________________

**Instructions:** Show all your work in order to receive proper credit. No formula sheets and no notes are allowed during the quiz. No cell phones, calculators, or any other electronic devices are allowed in a student's possession during any quiz. All such devices must be put away in the student's bag, out of reach of the student during the quiz. Quiz should be completed in one seating with no breaks. Box your final answer. Good luck!
**Timing:** 15 minutes

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1. **(4 pts)** The total surface area of a cube is changing at a rate of 12 $in^2/sec$ when the length of one of the sides is 10 in. At what rate is the volume of the cube changing at that time? You must include correct units as part of your answer.

**Solution:**

Let $x$ be the length of the side of the cube, let $A$ be the total surface area of the cube, let $V$ be the volume of the cube.
**Given:** $\frac{dA}{dt} = 12\ in^2/sec$ when $x = 10\ in.$

**Asked:** What is $\frac{dV}{dt}$ at the moment when $x = 10\ in.$ and $\frac{dA}{dt} = 12\ in^2/sec$

The equation for the volume of a cube is $V(x) = x^3$ and the equation for the total surface area of a cube is $A(x) = 6x^2$. Differentiating $V(x) = x^3$ with respect to time provides:

$$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$$

Although we know $x = 10$ in., we do not know $\frac{dx}{dt}$. We can simply find $\frac{dx}{dt}$ by differentiating $A(x) = 6 \cdot x^2$ with respect to time. Therefore, we obtain:

$$\frac{dA}{dt} = 12x \cdot \frac{dx}{dt}$$

By substituting the given information, we obtain:

$$12 = 12 \cdot 10 \cdot \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{10}$$

We computed $\frac{dx}{dt}$ as $\frac{1}{10}$ in./sec. Then we substitute this value and $x = 10$ in $\frac{dV}{dt}$ as follows:

$$\frac{dV}{dt} = 3 \cdot 10^2 \cdot \frac{1}{10} = 30$$

The volume of the cube is changing at a rate of 30 in$^3$/sec.

2. **(3 pts)** If $x$ units of a certain product are produced, the total cost is $C(x) = 3x + 9$ and the selling price per unit is $p(x) = 40 - 0.5x$. Determine the level of production that maximizes profit.

**Solution:**

Recall that $R(x) = p(x) \cdot x$. Therefore, $R(x) = (40 - 0.5x) \cdot x = 40x - 0.5x^2$.
Also recall that $P(x) = R(x) - C(x)$. The profit is maximized when $R'(x) = C'(x)$;

$$R'(x) = C'(x)$$

$$(40x - 0.5x^2)' = (3x + 9)'$$

$$40 - x = 3$$

$$x = 37$$

Thus, 37 items should be produced to maximize the profit.

3. **(3 pts)** If $x$ units of a certain product are produced, the total cost is $C(x) = x^3 - 22x^2 + 30x$. Find the level of production that minimizes the average cost. You must justify that your answer really does give the minimum average cost.

**Solution:** The average cost is

$$A(x) = \frac{C(x)}{x} = \frac{x^3 - 22x^2 + 30x}{x} = x^2 - 22x + 30$$

and $A(x)$ is minimized when $C'(x) = A(x)$. Thus,

$$C'(x) = A(x)$$

$$(x^3 - 22x^2 + 30x)' = x^2 - 22x + 30$$

$$3x^2 - 44x + 30 = x^2 - 22x + 30$$

$$2x^2 - 22x = 0$$

$$2x(x - 11) = 0$$

$$x = 0, 11$$

If we disregard the solution that suggest not producing any units ($x = 0$), it follows that the minimum average cost occurs when $x = 11$ units.

To verify that $x = 11$ units really gives the minimum average cost, find $A''(x)$ as:

$$A''(x) = (x^2 - 22x + 30)''$$

$$A''(x) = 2$$

Since $A''(x) > 0$, it follows that $x = 11$ really gives the minimum average cost.

Alternatively, we can construct a sign chart for $A'(x)$ by using the first-order critical number $x = 11$ and the endpoint of $x = 0$. We see that the function $A(x)$ is decreasing on the interval of $(0, 11)$ and increasing on the interval of $(11, +\infty)$. Therefore, we come to the same conclusion as the second-derivative test, which shows that $x = 11$ does really give the minimum average cost.