# Word Problems Practice

## Practice Final Exam #2

Q11) Use linear approximation to estimate $\sqrt[4]{78}$.

Solution: $L(x) = f(a) + f'(a)(x-a)$

$$f(x) \approx f(a) + f'(a)(x-a)$$

$$f'(a) \approx \frac{f(x) - f(a)}{x-a}$$

$\left[\begin{array}{l} a = ? \\ f(x) \end{array}\right]$

$a \to$ known value, compute w/out a calc.
close to desired value of 78.

$f(x) \to$ function $\quad f(x) = \sqrt[4]{x}$

$a \to 81$, $\quad f(a) = \sqrt[4]{81} = \sqrt[4]{3^4} = 3$

since $f(x) = \sqrt[4]{x} = (x^{\frac{1}{4}})$

$$f'(x) = \frac{1}{4} \cdot x^{\frac{1}{4}-1} = \frac{1}{4} \cdot x^{-3/4}$$

$$f'(a) = f'(81) = \frac{1}{4} \cdot (81^{-3/4}) = \frac{1}{4} \cdot (3^{\cancel{4}})^{-\frac{3}{\cancel{4}}}$$

$$f'(81) = \frac{1}{4} \cdot 3^{-3} = \frac{1}{4} \cdot \frac{1}{3^3} = \frac{1}{4} \cdot \frac{1}{27}$$

$$L(x) = f(a) + f'(a)(x-a)$$

$f(a) = f(81) = 3$

$f'(a) = f'(81) = \frac{1}{4} \cdot \frac{1}{27}$

$a = 81$

$$L(x) = 3 + \frac{1}{4} \cdot \frac{1}{27}(x-81)$$

when $x=78$, we can use $L(x)$ since we're close 81.

$$\sqrt[4]{78} \approx L(78) = 3 + \frac{1}{4} \cdot \frac{1}{27}(78-81)$$

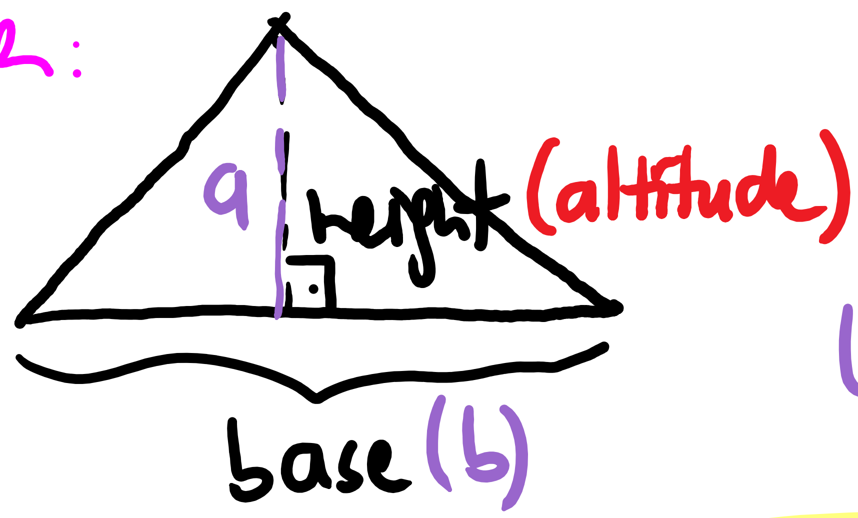
$$= 3 + \frac{1}{4} \cdot \frac{1}{\cancel{27}_{9}} \cancel{(-3)}^{-1}$$

$$\boxed{= 3 - \frac{1}{36}}$$

# Related Rates – Practice Final Exam #2

Q12) The altitude of a triangle is increasing at a rate of 1 ft/min. while the area is increasing at a rate 2 ft²/min.
At what rate is the base of the triangle changing when the altitude is 10 ft. and the area is 100 ft²?

Given:




$$\boxed{A = \frac{b \cdot a}{2}}$$ (Area of a triangle)

Let a be the altitude
b be the base

$$\boxed{\frac{da}{dt} = 1\frac{ft}{min}}, \quad \boxed{\frac{dA}{dt} = 2\frac{ft^2}{min}}$$

Asked: $\boxed{\frac{db}{dt} = ?}$ when $a = 10ft$, $A = 100ft^2$

$$\frac{dA}{dt} = \frac{d\left(\frac{b \cdot a}{2}\right)}{dt} = \left(\frac{b \cdot a}{2}\right)' = \frac{1}{2}(b \cdot a)'$$

$$\boxed{\frac{dA}{dt}} = \frac{1}{2}\left(\boxed{\frac{db}{dt}} \cdot a + \boxed{\frac{da}{dt}} \cdot b\right)$$

when $a = 10ft$, $A = 100 ft^2$,

$A = \frac{a \cdot b}{2}$

$\frac{100}{1} = \frac{10 \cdot b}{2}$

$b = 20\ ft$

$$\boxed{\frac{dA}{dt}} = \frac{1}{2}\left(\boxed{\frac{db}{dt}} \cdot a + \boxed{\frac{da}{dt}} \cdot b\right)$$

$\frac{dA}{dt} = 2 \frac{ft^2}{min}$, $\frac{db}{dt} = ?$

$a = 10 ft.$

$\frac{da}{dt} = 1 \frac{ft}{min}$

$b = 20 ft.$

$$\frac{2}{1} = \frac{1}{2}\left(\frac{db}{dt} \cdot 10 + 1 \cdot 20\right)$$

$$4 = 10 \cdot \frac{db}{dt} + 20$$

$$-20 \qquad\qquad -20$$

$$\frac{-16}{10} = \frac{10 \cdot \frac{db}{dt}}{10}$$

$$\boxed{-1.6 \frac{ft}{min} = \frac{db}{dt}}$$

The base is decreasing at a rate of $1.6 \frac{ft}{min}$.

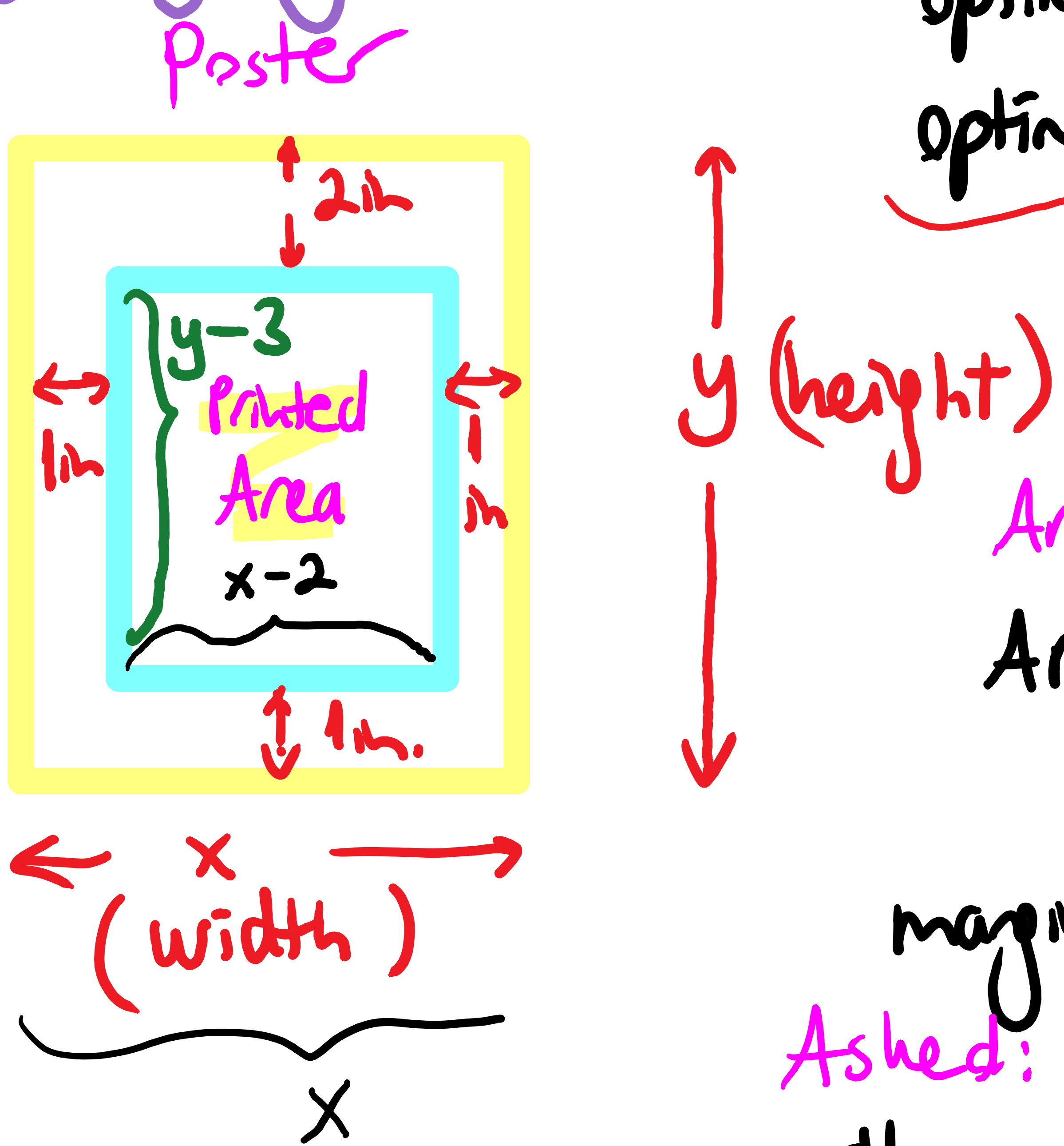
The rate of change of base w/ respect to time is $-1.6\ ft/min$

# Second Exam (Sample) - Q5

A poster is to have a total area of 150 in$^2$, which includes a central printed area, 1-inch margins at the bottom and sides, and a 2-inch margin at the top. What poster dimensions (in inches) will give the largest printed area? Use calculus to justify your answer.

optimal width: $x =$ ______

optimal height: $y =$ ______




Given:

Area of the poster is 150 in$^2$

Area of the poster ⇒

$x \cdot y = 150 \text{ in}^2$ Constraint

margins are given

Asked: What dimensions will give the largest (MAX) printed area.

Area of a printed area ⇒ width · height

obj. f: $(x-2) \cdot (y-3)$

Objective function: $A(x,y) = (x-2)\cdot(y-3)$

Constraint: $\frac{x\cdot y}{x} = \frac{150}{x}$ in$^2$ $\Rightarrow$ $y = \frac{150}{x}$

$$\underbrace{A\left(x, \frac{150}{x}\right)} = (x-2)\cdot\left(\frac{150}{x} - 3\right) \quad \text{MAX.}$$

$$A(x) = \cancel{x}\cdot\frac{150}{\cancel{x}} - 3x - 2\cdot\frac{150}{x} + 6$$

$$= 150 - 3x - \frac{300}{x} + 6$$

$$A(x) = 156 - 3x - \frac{300}{x} \rightarrow (-300\cdot x^{-1}) \quad \text{MAX}$$

$A'(x) = 0$ or DNE to find first-order critical #s

$$A'(x) = -3 + 300\cdot(+1)\cdot x^{-2}$$

$$A'(x) = -3 + 300\cdot x^{-2}$$

$$A'(x) = 0 = -3 + 300\cdot x^{-2} \Rightarrow 0 = -3 + \frac{300}{x^2}$$

$$\frac{3}{1} = \frac{300}{x^2} \Rightarrow 3x^2 = 300$$

$$x^2 = 100$$

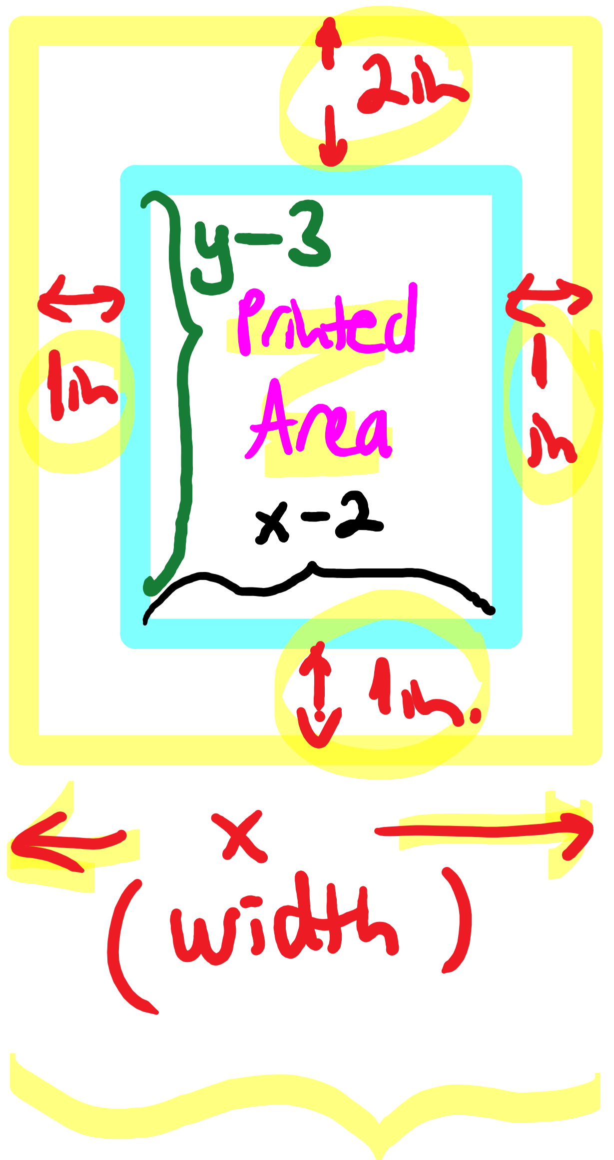
$$x = \mp 10$$

$$x = 10 \text{ in.}$$

$A'(x)$ DNE

$$A'(x) = -3 + \frac{300}{x^2} \Rightarrow x = 0$$

Poster




$y$ (height)

Constraint: $x \cdot y = 150 \text{ in}^2$

$A(x)$

$x \uparrow \quad y \downarrow$

min. value of $x$ is 2 in.

max. value of $x$ $\boxed{?}$ 50 in.

↳ depends on the min. value of $y$

(min. value of $y$ is 3 in.)

Since the min. value of $y = 3$ in. $(y \geq 3)$

$\underbrace{x \cdot y = 150}$ $\Rightarrow$ $x \cdot 3 = 150$

$y = \dfrac{150}{x} \geq 3$ $\qquad \dfrac{150}{3} \geq x$

$50 \geq x$

Domain of $x$ is: $[2, 50]$

Use calculus to justify your answer

($x = 10$ m. as the first-order critical #)

~~$x = 0$ m.~~ $x = 10$ m.

domain for $x$: $[2, 50]$

method #1

sign chart for $A'(x) = -3 + \dfrac{300}{x^2}$

| | 2 → 5 → 10 | 10 → 30 → 50 |
|---|---|---|
| sign of $A'(x)$ | + | − |
| incr/decr. | incr. | decr. |

MAX AREA

max at $x = 10$.

Test points:

$A'(5) = -3 + \dfrac{300}{25} = -3 + 12 > 0$

$A'(30) = -3 + \dfrac{\cancel{300}^{10}}{\cancel{30} \cdot 30} = -3 + \dfrac{1}{3} < 0$

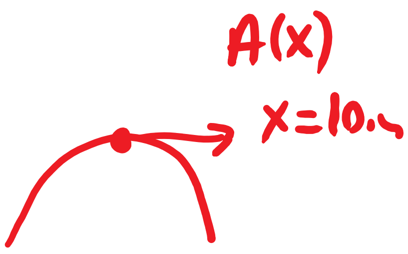
method #2

Test concavity for $A''(x)$

$A'(x) = -3 + 300 \cdot x^{-2}$

$A''(x) = 300(-2) \cdot x^{-3} = \dfrac{-600}{x^3}$ → neg. / → pos.

A''(x) is negative in its domain:



Method #3 $A(x) = (x-2)\cdot\left(\frac{150}{x} - 3\right)$

Endpoints: $x=2, 50$

First-order critical #: $x=10$

$A(2)=0$

$A(50) = (50-2)\cdot\left(\cancel{\frac{150}{50} - 3}\right) \overset{0}{=} 0$

$A(10) = (10-2)\cdot\left(\frac{150}{10} - 3\right) = 8\cdot(15-3) = 8\cdot 12$

$= \boxed{96\ in^2.}$

When $x=10$ in. $\Rightarrow x\cdot y = 150\ in^2 \Rightarrow y = 15$ in.

$A(x,y) = (x-2)\cdot(y-3)$

dimensions for the poster that gives the largest area are:

$\boxed{\begin{array}{l} x-2 = 8\text{ in.} \\ y-3 = 15-3 = 12\text{ in.} \end{array}}$