MIDTERM #2 REVIEW

1) Estimate $\frac{1}{\sqrt[3]{29}}$

$$L(x) = f(a) + f'(a)(x-a)$$

$a \to 27$, $f(a) = f(27) = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$

$$f(x) = \frac{1}{\sqrt[3]{x}} = x^{-1/3}$$

$$f'(x) = -\frac{1}{3} \cdot x^{-4/3}$$

$$f'(27) = -\frac{1}{3} \cdot (27)^{-4/3} = -\frac{1}{3} \cdot (3^{\cancel{3}})^{-\frac{4}{\cancel{3}}}$$

$$= -\frac{1}{3} \cdot 3^{-4} = -\frac{1}{3} \cdot \frac{1}{3^4}$$

$$L(x) = \frac{1}{3} + \left(-\frac{1}{3^5}\right)(x-27)$$

$$L(29) = \frac{1}{3} + \left(-\frac{1}{3^5}\right)\underbrace{(29-27)}_{2} = \frac{1}{\underset{(3^4)}{3}} - \frac{2}{3^5}$$

$$= \frac{81}{243} - \frac{2}{243} = \frac{79}{243}$$

$$\boxed{L(29) = \frac{79}{243}}$$

Q2) $\lim_{x\to 0}\left(\frac{1-\cos(4x)}{x^2}\right)$

$\overset{\text{"DSP"}}{=} \frac{1-\cos 0}{0^2} = \text{"}\frac{0}{0}\text{"}$

$\overset{LR}{=} \lim_{x\to 0} \frac{4\cdot\sin(4x)}{2x} \overset{\text{"DSP"}}{=} \frac{4\cdot\sin 0}{2\cdot 0} = \text{"}\frac{0}{0}\text{"}$

$\overset{LR}{=} \lim_{x\to 0} \frac{4\cdot 4\cdot\cos(4x)}{2} \overset{\text{"DSP"}}{=} \frac{4\cdot 4\cdot 1}{2} = \boxed{8}$

Q3) $\lim\limits_{x \to 1^-} \left((1-x) \cdot \sec\left(\frac{\pi x}{2}\right) \right)$

$$\sec x = \frac{1}{\cos x}$$

$$\lim_{x \to 1^-} \left(\frac{(1-x)}{\cos\left(\frac{\pi x}{2}\right)} \right) \overset{\text{"DSP"}}{=} \frac{1-1}{\cos \frac{\pi}{2}} = \text{"}\frac{0}{0}\text{"}$$

$$\overset{\text{"LR"}}{=} \lim_{x \to 1^-} \frac{-1}{-\sin\left(\frac{\pi x}{2}\right) \cdot \frac{\pi}{2}} \overset{\text{"DSP"}}{=} \frac{-1}{-\sin\left(\frac{\pi}{2}\right) \cdot \frac{\pi}{2}} = \frac{\cancel{-}1}{\cancel{-}1 \cdot \frac{\pi}{2}}$$

$$= \boxed{\frac{2}{\pi}}$$

Q4) $\lim_{x\to 0^+} (1+\sin(3x))^{\frac{10}{x}}$ "OSP" $(1+\sin 0)^{\frac{10}{0}}$

1^∞

Let $L = \lim_{x\to 0^+} (1+\sin(3x))^{\frac{10}{x}}$

$$\ln L = \ln\left(\lim_{x\to 0^+} (1+\sin(3x))^{\frac{10}{x}}\right)$$

$$= \lim_{x\to 0^+} \left(\ln (1+\sin(3x))^{\frac{10}{x}}\right)$$

$$\ln L = \lim_{x\to 0^+}\left(\frac{10\cdot \ln(1+\sin(3x))}{x}\right)$$

$$\overset{\text{"OSP"}}{=} \frac{10\cdot \ln 1}{0} = \text{"}\frac{0}{0}\text{"}$$

$$(\ln(u))' = \frac{u'}{u}$$

$$\ln L \overset{\text{"LR"}}{=} \lim_{x\to 0^+}\left(\frac{10\cdot\frac{3\cdot\cos 3x}{1+\sin(3x)}}{1}\right)$$

$$\overset{\text{"OSP"}}{=} 10\cdot\frac{3\cdot\cos(0)}{1+\sin 0} = 30$$

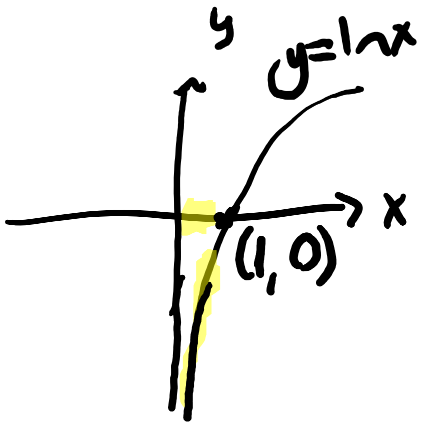
Last step:

$\ln_e L = 30 \Rightarrow \boxed{L = e^{30}}$

Q5) Find the x-coordinates of all local extremum for $f(x)=4x^2\cdot\ln x-2x^2$

$f'(x)=0$ or DNE

none



$$f'(x)=8x\cdot\ln x+4x^2\cdot\frac{1}{x}-4x$$

$$=8x\cdot\ln x+4x-4x=8x\cdot\ln x$$

$$f'(x)=0=8x\cdot\ln x \Rightarrow 8x=0 \quad \text{or} \quad \ln x=0$$

$x=0$

$x=e^0=1$

sign chart for $f'(x)$

	0 ($\frac{1}{2}$) 1	1 (e^3)
sign of $f'(x)$	$-$	$+$
incr/decr.	↘	↗

testpoints: $x=\frac{1}{2}, e^3$

$$f'\left(\tfrac{1}{2}\right)=8\cdot\tfrac{1}{2}\cdot\ln\left(\tfrac{1}{2}\right)$$

$$=4\cdot\ln(2^{-1})<0$$

$$f'(e^3)=8\cdot e^3\cdot\ln(e^3)>0$$

local min at $x=1$

No local max!

Q6) Find ALL Horizontal Asymptotes for

$$f(x) = \frac{12x+5}{\sqrt{16x^2+x+1}}$$

$$\lim_{x \to +\infty} \frac{12x+5}{\sqrt{16x^2+x+1}}$$ → find the highest exponent term "x^2" then div. ALL by $\sqrt{x^2} = |x|$

if $x>0$ x

$\boxed{\text{if } x<0 \quad -x}$

$$\lim_{x \to \infty} \frac{\frac{12x+5}{x}}{\sqrt{\frac{16x^2+x+1}{x^2}}}$$

$$= \lim_{x \to \infty} \frac{12+\frac{5}{x}}{\sqrt{16+\frac{1}{x}+\frac{1}{x^2}}} = \lim_{x \to \infty} \frac{12}{\sqrt{16}} = \frac{12}{4} = 3$$

0 0 0

$\boxed{y=3}$

$$\lim_{x \to -\infty} \frac{12x+5}{\sqrt{16x^2+x+1}} = \lim_{x \to -\infty} \frac{\frac{12x}{x}+\frac{5}{x}}{-\sqrt{\frac{16x^2}{x^2}+\frac{x}{x^2}+\frac{1}{x^2}}}$$

0 0 0

$$= \lim_{x \to -\infty} \frac{12+0}{-\sqrt{16+0+0}} = \frac{12}{-4} = -3$$

$\boxed{y=-3}$

The H.A. of $f(x)$ are: $y=3$, $y=-3$

Q7) $C(x) = x + 20$ (cost)

$p(x) = \dfrac{35-x}{5+x}$ (price per item)

Determine the level of production that maximizes profit. (find x)

$P(x) = R(x) - C(x)$, $R(x) = p(x) \cdot x$

max $P(x) \to P'(x) = 0$ or DNE

$$P'(x) = \underbrace{R'(x) - C'(x) = 0}_{R'(x) = C'(x)} \quad \text{or DNE}$$

$$R(x) = \frac{35-x}{5+x} \cdot x = \frac{35x - x^2}{5+x}$$

To max. $P(x) \to R'(x) = C'(x)$

$$R'(x) = \left(\frac{35x - x^2}{5+x} \right)' = \frac{(35-2x)(5+x) - (35x - x^2) \cdot 1}{(5+x)^2}$$

$$= \frac{35 \cdot 5 + \cancel{35x} - 10x - 2x^2 - \cancel{35x} + x^2}{(5+x)^2}$$

$$= \frac{175 - 10x - x^2}{(5+x)^2}$$

$C'(x) = (x+20)' = 1$

$$R'(x) = \frac{175-10x-x^2}{(5+x)^2}$$

$$C'(x) = (x+20)' = 1$$

max $P(x)$

$R'(x) = C'(x)$

$$\frac{175-10x-x^2}{(5+x)^2} = \frac{1}{1} \Rightarrow 175-10x-x^2 = x^2+10x+25$$

$$2x^2+20x-150=0$$

$$2(x^2+10x-75)=0$$

$15 \quad -5$

$$2(x+15)(x-5)=0$$

$\cancel{x=-15}$, $\boxed{x=5}$