Math 135, Quiz #10 Solutions

Name: ____________________ Section: ____________________

Instructions: Show all your work in order to receive proper credit. No formula sheets and no notes are allowed during the quiz. No cell phones, calculators, or any other electronic devices are allowed in a student's possession during any quiz. All such devices must be put away in the student's bag, out of reach of the student during the quiz. Quiz should be completed in one seating with no breaks. Box your final answer. Good luck!
Timing: 15 minutes

1. **(3 pts)** Find the antiderivative. $\int (2e^x - 3\cos x + \sqrt[4]{x})dx$

Solution: By using the rules of antiderivatives we obtain:
$\int (2e^x - 3\cos x + \sqrt[4]{x})dx = 2e^x - 3\sin x + \frac{4}{5} \cdot x^{5/4} + C$

2. **(3 pts)** A particle travels along the x-axis in such a way that its acceleration at time t is $a(t) = 3\sqrt{t} + t^3$. If it starts with an initial velocity of 1 (that is, $v(0) = 1$), determine its velocity when $t = 4$.

Solution:
Recall that: $v(t) = \int (a(t)dt$
$v(t) = \int (3t^{\frac{1}{2}} + t^3)dt = 2t^{3/2} + \frac{t^4}{4} + C$
By using the initial value of $v(0) = 1$, we obtain $C = 1$.
Therefore, the velocity of the particle is:

$$v(t) = 2t^{3/2} + \frac{t^4}{4} + 1$$

The velocity of the particle at $t = 4$ is:

$$v(4) = 2 \cdot 4^{3/2} + \frac{4^4}{4} + 1 = 2 \cdot 2^3 + 4^3 + 1 = 16 + 64 + 1 = 81$$

3. **(4 pts)** Estimate the area under the graph of $f(x) = x^2 + 4x$ and above the x-axis on the interval $[0, 1]$ by using Riemann sum with right endpoints and 4 rectangles. **Do not simplify your final answer.**

Solution: The width of each rectangle is: $\Delta x = \frac{1-0}{4} = \frac{1}{4}$

Rectangle#	Right-endpoint (x)	Height (y)
1	$0 + \frac{1}{4}$	$\frac{17}{16}$
2	$\frac{1}{4} + \frac{1}{4}$	$\frac{9}{4}$
3	$\frac{2}{4} + \frac{1}{4}$	$\frac{57}{16}$
4	$\frac{3}{4} + \frac{1}{4}$	5

The total estimated area is: $A = \frac{1}{4} \cdot \left(\frac{17}{16} + \frac{9}{4} + \frac{57}{16} + 5 \right)$

4. **(Extra Credit - 1 pt)** Calculate the exact area under the graph of $f(x) = x^2 + 4x$ and above the x-axis on the interval $[0, 1]$. **Simplify your final answer as much as possible.**

Solution: The exact area is computed by the definite integral:

$$\int_0^1 (x^2 + 4x)\ dx = \left(\frac{x^3}{3} + \frac{4x^2}{2} \right) \Bigg|_0^1 = \left((\frac{1}{3} + 2 \cdot 1^2) - (0 + 0) \right) = \left(\frac{1}{3} + 2 \right) = \frac{7}{3}$$