# Final Review — Ch 5, 4

Exp) Evaluate $\displaystyle\int_{-\ln 2}^{\ln 2} \frac{1}{2}(e^{x}-e^{-x})\cdot dx$

$$\frac{1}{2}\int_{-\ln 2}^{\ln 2}(e^{x}-e^{-x})\cdot dx$$

$$\int e^{x}\cdot dx = e^{x}$$

$$\int -e^{-x}\cdot dx = e^{-x}$$

$$\frac{d}{dx}(e^{-x}) = e^{-x}\cdot(-1)$$

$$\frac{1}{2}\left(e^{x}+e^{-x}\right)\Big|_{-\ln 2}^{\ln 2}$$

$$\frac{1}{2}\left[e^{\ln 2}+e^{-\ln 2}-\left(e^{-\ln 2}+e^{+(+\ln 2)}\right)\right]$$

$$\frac{1}{2}\left[\cancel{e^{\ln 2}}+\cancel{e^{-\ln 2}}-\cancel{e^{-\ln 2}}-\cancel{e^{\ln 2}}\right]$$

$$\underbrace{e^{\ln 2}=2}$$

$$\frac{1}{2}\cdot 0 = 0$$

Exp2) Evaluate $\int_{-1}^{2} (x+|x|) \cdot dx$

Recall:

$$f(x)=|x| \begin{cases} x \geq 0 & f(x)=x \quad ① \\ x<0 & f(x)=-x \quad ② \end{cases}$$

② ①

$$\int_{-1}^{0} (x+|x|) \cdot dx + \int_{0}^{2} (x+|x|) \cdot dx$$

$(x+(-x))$

$$\int_{-1}^{0} 0 \cdot dx + \int_{0}^{2} (x+x) \cdot dx = 0 + \int_{0}^{2} 2x \cdot dx$$

$$= 0 + x^2 \Big|_{0}^{2}$$

$$= 2^2 - 0^2 = \boxed{4}$$

Exp3)

$$f(x) = \begin{cases} 5 & \text{for } -3 \le x < -1 \quad ① \\ 4-x & \text{for } -1 \le x < 2 \quad ② \\ 2x-2 & \text{for } 2 \le x \le 5 \quad ③ \end{cases}$$

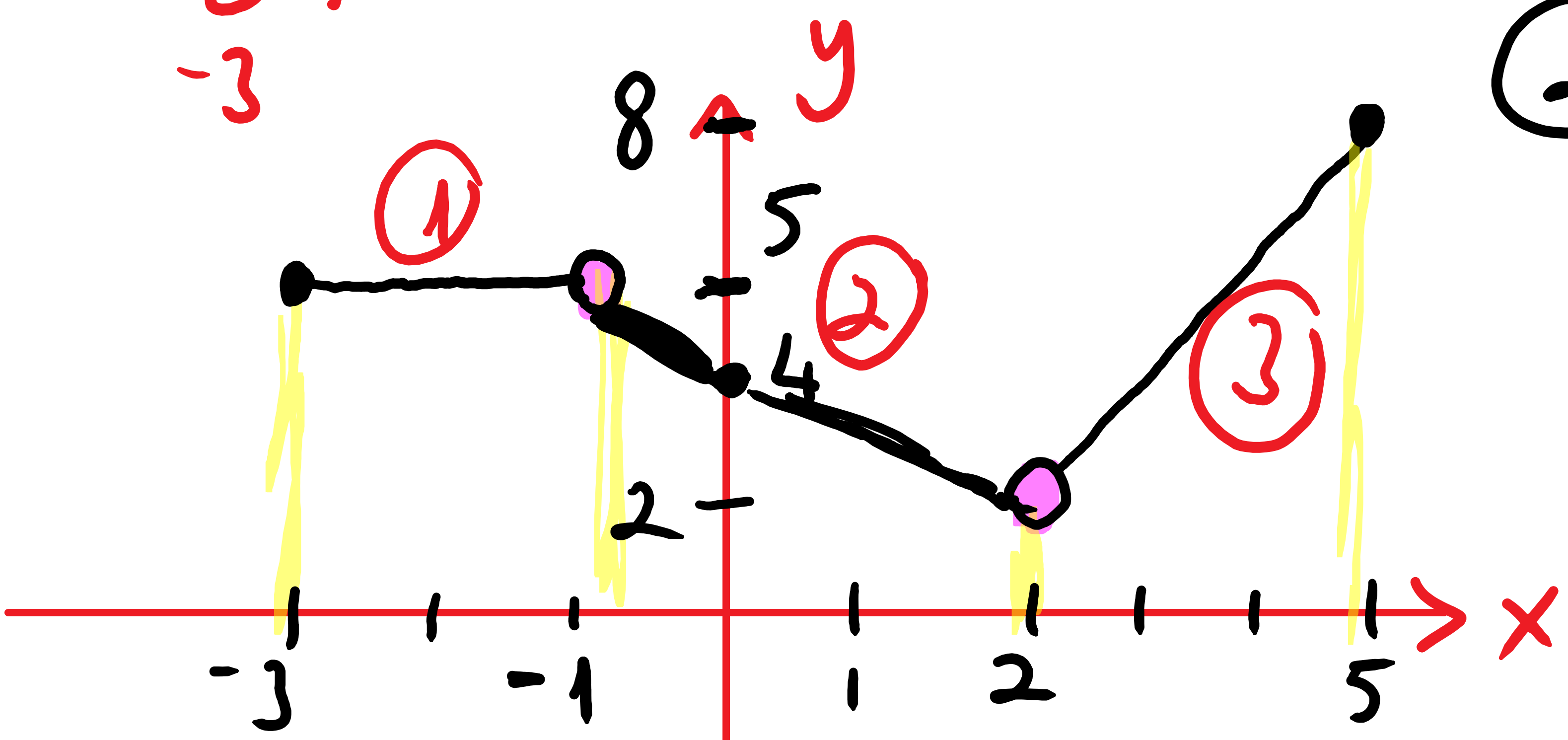
① Sketch the graph of $f$ on $[-3, 5]$

② Show that $f(x)$ is continuous on $[-3, 5]$

③ $\displaystyle\int_{-3}^{5} f(x) \cdot dx = ?$

\ (−)

/ (+)




② $y = 4 - x$

$m = -1$

$b = 4$

$x = 2$

$y = 4 - x$
$= 2$

$(2, 2)$

③ $x = 2$ $\quad f(2) = 2 \cdot 2 - 2 = 2$

$f(5) = 2 \cdot 5 - 2 = 8$

② Check for continuity

$$\lim_{x \to -1^-} (5) = \lim_{x \to -1^+} (4-x) = f(-1)$$

$$5 = \underbrace{(4 - -1)} = 5 \checkmark$$

$$\lim_{x \to 2^-} (4-x) = \lim_{x \to 2^+} (2x-2) = f(2)$$

$$4-2 = 2 \cdot 2 - 2 = (2 \cdot 2 - 2)$$

$$2 = 2 = f(2) \checkmark$$

$$\int_{-3}^{5} f(x) \cdot dx = \int_{-3}^{-1} 5 \cdot dx + \int_{-1}^{2} (4-x) + \int_{2}^{5} (2x-2) \cdot dx$$

$$= 5x \Big|_{-3}^{-1} + \left(4x - \frac{x^2}{2}\right) \Big|_{-1}^{2} + (x^2 - 2x) \Big|_{2}^{5}$$

$$= 5\left(-1-(-3)\right) + \left(4\cdot 2 - \frac{2^2}{2} - \left(4\cdot(-1) - \frac{(-1)^2}{2}\right)\right) + \left(25 - 10 - (4-4)\right)$$

$$= 5(-1+3) + \left(8 - 2 - \left(-4 - \frac{1}{2}\right)\right) + (15 - 0)$$

$$= 5\cdot 2 + \left(\underbrace{8 - 2}_{6} + 4 + \frac{1}{2}\right) + 15$$

$$6 \quad 10$$

$$= \underbrace{10 + 10.5}_{20.5} + 15 = 35.5$$

Exp) Estimate the area under the graph of $f(x) = x + \sin x$ and over the x-axis on $[0, \pi]$ by using Riemann sum right edpoints with $n=4$.

$$\int_0^{\pi} (x + \sin x)\, dx \quad \Big\} \text{ exact value}$$

$\Delta x = \dfrac{b-a}{n}$

$\Delta x = \dfrac{\pi - 0}{4}$

$\underbrace{\Delta x = \dfrac{\pi}{4}}_{\text{width}}$

| $x$ | $f(x)$ - height of rectangles |
|---|---|
| ① $0 + \frac{\pi}{4}$ | $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4} + \sin\frac{\pi}{4} = \frac{\pi}{4} + \frac{\sqrt{2}}{2}$ |
| ② $\frac{\pi}{4} + \frac{\pi}{4}$ | $f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \sin\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + 1$ |
| ③ $\frac{2\pi}{4} + \frac{\pi}{4}$ | $f\left(\frac{3\pi}{4}\right) = \frac{3\pi}{4} + \sin\left(\frac{3\pi}{4}\right) = \frac{3\pi}{4} + \frac{\sqrt{2}}{2}$ |
| ④ $\frac{3\pi}{4} + \frac{\pi}{4}$ = $\pi$ | $f(\pi) = \pi + \sin(\pi) = \pi + 0$ |

$\frac{\pi}{4}\left(\frac{5\pi}{2} + \sqrt{2} + 1\right) \rightarrow$

$$\text{Area} = w \cdot h = \frac{\pi}{4}\left(\frac{\pi}{4} + \frac{\sqrt{2}}{2} + \frac{\pi}{2} + 1 + \frac{3\pi}{4} + \frac{\sqrt{2}}{2} + \pi\right)$$

$\pi$

$$\text{Area} = w \cdot h = \frac{\pi}{4}\left(\frac{\pi}{4} + \frac{\sqrt{2}}{2} + \frac{\pi}{2} + 1 + \frac{3\pi}{4} + \frac{\sqrt{2}}{2} + \pi\right)$$

Exp) Find $F'(x)$ when $F(x) = \int_{-1}^{x} \frac{t^5}{3+t^6} \cdot dt$

$$F'(x) = \frac{x^5}{3+x^6}$$

Exp) Find relative (local) min/max for $f(x) = (x^2 - 3x + 1) \cdot e^{-x}$

$$f'(x) = (2x-3) \cdot e^{-x} + (x^2 - 3x + 1) \cdot (-1) \cdot e^{-x}$$

$$= -e^{-x}(-2x + 3 + x^2 - 3x + 1)$$

$$= -e^{-x}(x^2 - 5x + 4)$$

$$-4 \quad -1$$

$$= -e^{-x}(x-4)(x-1) = \frac{-(x-4)(x-1)}{e^x}$$

1st order critical #s:

$f'(x) = 0$ or $\underbrace{\text{DNE}}_{\text{none}}$ $\quad x = 4, \; x = 1$

$f'(x) = -e^{-x}(x-4)(x-1)$ $\qquad x = 4, 1$

sign chart for $f'(x)$

| | $-\infty$ | 0 | 1 | 2 | 4 | 10 | $+\infty$ |
|---|---|---|---|---|---|---|---|
| sign of $f'(x)$ | | $-$ | | $+$ | | $-$ | |
| incr/ decr. | | | local min at $x=1$ | | local max at $x=4$ | | |

testpoints:

$x=0 \quad f'(0) = -\underbrace{e^{-0}}_{1}\underbrace{(0-4)(0-1)}_{+} = (-)(+) = -$

$x=2 \quad f'(2) = -\overset{1}{\cancel{e^{-2}}}(\underbrace{2-4}_{-})\overset{+}{}(\underbrace{2-1}_{+}) = (-)(-)(+) = +$

$x=10 \quad f'(10) = -e^{-10}(\underbrace{10-4}_{+})(\underbrace{10-1}_{+}) = (-)(+)(+) = -$

$f''(c) < 0 \quad \cap \quad c \rightarrow$ local MAX

$f''(c) > 0 \quad \cup \quad c \rightarrow$ local MIN

Exp) Find the limit:

$$\lim_{x \to -\infty} \frac{3x}{\underbrace{\sqrt{4x^2+10}}_{\text{highest exponent of denom is } \sqrt{x^2}}} \rightarrow \sqrt{x^2} = |x|$$

$x \geq 0 \quad \sqrt{x^2} = |x| = x$

$x < 0 \quad \sqrt{x^2} = |x| = -x$

$$\lim_{x \to -\infty} \frac{\frac{3x}{x}}{\sqrt{\frac{4x^2}{x^2} + \frac{10}{x^2}}} \quad 3$$

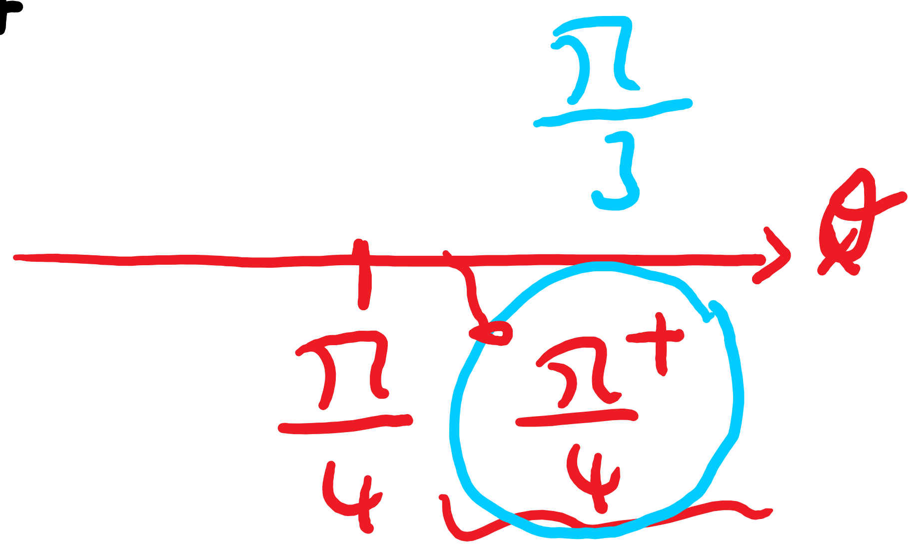
$$\underbrace{|-2|}_{x} = -(-2) = 2$$

$$\lim_{x \to -\infty} \frac{\cancel{\frac{3x}{x}}}{-2} = \lim_{x \to -\infty} \frac{3}{-2} = \frac{-3}{2}$$

Exp) $\lim_{x\to\frac{\pi}{4}^+} \dfrac{\sec x}{\tan x - 1} = \lim_{x\to\frac{\pi}{4}^+} \dfrac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x} - 1}$

$$= \lim_{x\to\frac{\pi}{4}^+} \dfrac{\frac{1}{\cos x}}{\frac{\sin x - \cos x}{\cos x}} = \lim_{x\to\frac{\pi}{4}^+} \dfrac{1}{\underbrace{\sin x - \cos x}_{0^+}} = +\infty$$

$$\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$



$$\lim_{x\to\frac{\pi}{4}^-} \dfrac{1}{\underbrace{\sin x - \cos x}_{0^-}} = -\infty$$

$$\sin\frac{\pi}{3} - \cos\frac{\pi}{3}$$

$$\frac{\sqrt{3}}{2} - \frac{1}{2} > 0$$

Exp) $\lim_{x \to 0} \frac{x^2+\sin(x^2)}{x^2+x^3}$ $=$ "DSI" $\frac{0+\sin 0}{0+0} = \frac{0}{0}$

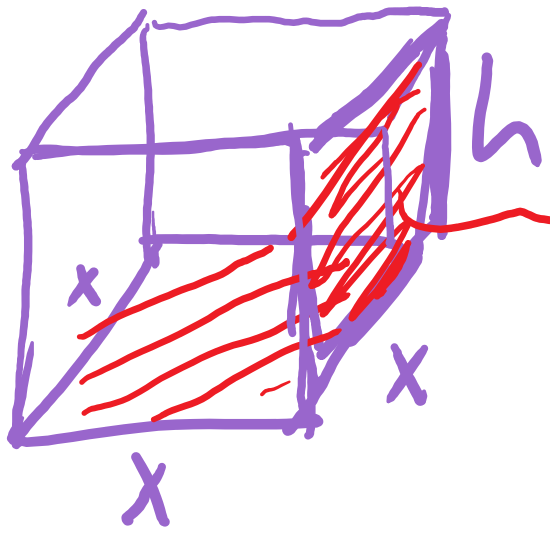
LR

$= \lim_{x\to 0} \frac{2x+2x\cdot\cos(x^2)}{2x+3x^2}$

$= \lim_{x\to 0} \frac{\cancel{x}(2+2\cos(x^2))}{\cancel{x}(2+3x)}$ "DSP" $= \frac{2+2\cdot\cos 0}{2+3\cdot 0}$

$= \frac{2+2}{2+0} = \boxed{2}$

Exp) 4.6 Q 34 on Text.




$A = x \cdot h$


$\phantom{Cost \quad \$1/ft^2 \times} \; x^2 + 4 \cdot x \cdot h$

Cost $\quad \$1/ft^2 \times \text{base} + \text{sides}$

$\quad \$5/ft^2 \times \text{top} \quad x \cdot x$

Goal: $\underbrace{\text{MAX Volume}}$ $\qquad$ $\underline{\text{Cost} = \$72}$

Obj. F: $V = x \cdot x \cdot h = x^2 h \rightarrow \text{MAX}$

Total Cost $\Rightarrow 72 = 1 \cdot (x^2 + 4xh) + 5(x^2)$

$$72 = x^2 + 4xh + 5x^2$$

$$72 = 6x^2 + 4xh \quad \text{Constraint}$$

$$\boxed{\frac{72 - 6x^2}{4x} = h}$$

$$V(x) = hx^2 = \left(\frac{72-6x^2}{4x}\right)\cdot x^2$$

$$= \frac{(72-6x^2)\cdot x}{4} = \frac{72x-6x^3}{4}$$

$V'(x) = 0$ or DNE

$$V'(x) = \frac{1}{4}\left(72 - 18x^2\right) = \underbrace{18 - \frac{9}{2}x^2}_{} = 0$$

$x = \pm 2$ $\quad \cancel{x=-2}$ $\quad \boxed{x = 2\text{ ft.}}$ $\quad \boxed{x \geq 0}$

$$V''(x) = -\frac{9}{2}\cdot 2x = -9x$$

$< 0 \rightarrow$ max

$\boxed{x = 2\text{ ft.} \quad h = 6\text{ ft.}}$