Algebra Review

* Simplify Exponents
* Simplify & solve algebraic equations & inequalities
* Functions (domain, composition, piece-wise)
* Equations of Lines

Exp1) Simplify and express your answer with positive exponents only (x, y both non-zero)

$$\frac{-72 \cdot x^{-19} \cdot y^8}{(x^4 y^{-3})^2 \cdot (-3)^2 \cdot \sqrt[3]{x^6}} = \frac{-72 \cdot x^{-19} \cdot y^8}{x^8 \cdot y^{-6} \cdot 9 \cdot x^{6/3}}$$

$$= \frac{-72}{9} \cdot \frac{x^{-19}}{x^8 \cdot x^2} \cdot \frac{y^8}{y^{-6}}$$

$$= -8 \cdot \frac{x^{-19}}{x^{10}} \cdot y^{8-(-6)}$$

$$= -8 \cdot x^{-29} \cdot y^{14}$$

$$= \boxed{\frac{-8y^{14}}{x^{29}}}$$

Recall:

$x^0 = 1 \quad (x \neq 0)$

$(x^m)^n = x^{m \cdot n}$

$x^{-m} = \frac{1}{x^m}$

Exp2) Simplify, write the answer with positive exponents only

a) $-81^{1/4} = -(3^4)^{1/4} = -(3^{4 \cdot \frac{1}{4}}) = -3$

b) $[(-3)^4]^{\frac{1}{4}} = (3^4)^{1/4} = 3$

c) $-\sqrt[4]{81} = -\sqrt[4]{3^4} = -3^{\frac{4}{4}} = -3$

d) $\sqrt[4]{-81} \Rightarrow$ Not a real number (not in scope of this class)

Exp3) Solve the equation

a) $3 - \frac{1}{t} = \frac{2}{t^2} \quad (t \neq 0)$

$$\frac{3}{\underset{(t^2)}{1}} - \frac{1}{\underset{(t)}{t}} = \frac{2}{\underset{(1)}{t^2}} \Rightarrow t^2 \cdot \frac{3t^2 - t}{t^2} = \frac{2}{t^2} \cdot t^2$$

$$3t^2 - t = 2$$

$$3t^2 - t - 2 = 0 \Rightarrow \underbrace{(3t+2)}\cdot\underbrace{(t-1)} = 0$$

$3t \quad 2$

$t \quad -1$

$-3t + 2t = -t$ ✓

$3t+2 = 0 \qquad t - 1 = 0$

$$\boxed{t = -2/3, \quad t = 1}$$

Exp4) Solve the equation:

$$x(2x-5) = (x-1)^2$$

Recall:

$$(x-1)^2 \neq x^2 - 1^2$$

$$(x-1)^2 = (x-1)(x-1)$$

$$= x^2 - 2x + 1$$

$$x(2x-5) = (x-1)^2$$

$$2x^2 - 5x = x^2 - 2x + 1$$

$$2x^2 - 5x - x^2 + 2x - 1 = 0$$

$$x^2 - 3x - 1 = 0$$

$$1x^2 - 3x - 1 = 0$$

$a = 1,\ b = -3,\ c = -1$

Recall:

$$ax^2 + bx + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{3 \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1}$$

$$x_{1,2} = \frac{3 \pm \sqrt{9+4}}{2}$$

Solution: $x_1 = \dfrac{3+\sqrt{13}}{2}$, $x_2 = \dfrac{3-\sqrt{13}}{2}$

Exp6) Find all real solutions to: $4x^{7/2} - x^{5/2} - 3x^{3/2} = 0$

$$4x^{7/2} - x^{5/2} - 3x^{3/2} = 0$$

$$x^{3/2}\left(4x^{4/2} - x^{2/2} - 3\right) = 0$$

$$x \cdot x^{1/2}\left(4x^2 - x - 3\right) = 0$$

$$\begin{array}{cc} 4x & 3 \\ x & -1 \end{array}$$

$$\underbrace{x}_{0} \cdot \underbrace{\sqrt{x}}_{0} \cdot \underbrace{(4x+3)}_{0}\underbrace{(x-1)}_{0} = 0$$

$x = 0$

$\sqrt{x} = 0$

$4x+3 = 0 \Rightarrow x = -3/4$

$x - 1 = 0 \Rightarrow x = 1$

check for extraneous solutions

$\Rightarrow \sqrt{-3/4}$ not a real number!

$\sqrt{x}$ is defined only for $x \geq 0$

The solutions are: $x = 0$, $x = 1$

Exp7) Simplify the difference quotient; $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$

for: $f(x) = \sqrt{x}$

$f(x) = \sqrt{x}$

$f(x+h) = \sqrt{x+h}$

$$\frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

Recall:

$(a-b)(a+b) = a^2 - b^2$

$$\frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \boxed{\frac{1}{\sqrt{x+h} + \sqrt{x}}}$$

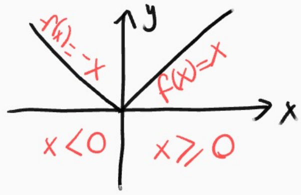
Exp8) Find the composite functions for: $f(x) = 5x + 7$, $g(x) = x^2$

a) $(f \circ g)(x) = f(g(x)) = 5x^2 + 7$

b) $(g \circ f)(x) = g(f(x)) = (5x+7)^2$

c) $(g \circ g)(x) = g(g(x)) = (x^2)^2 = x^4$

A famous piece-wise function (Absolute Value F.)



$$f(x)=|x| \begin{cases} \text{if } x\geq 0 & f(x)=x \\ \text{if } x<0 & f(x)=-x \end{cases}$$

Recall: $f(x)=\sqrt{x^2} \neq x$

$f(x)=\sqrt{x^2}=|x|$

E.g: Simplify $\sqrt{ax^2+bx^2} = \sqrt{x^2(a+b)}$

$= |x|\ (a+b)$

Exp 6) Find the equation of the following lines:

a) through $(-2, 6)$ with slope 5

$y-y_1=m(x-x_1)$ point-slope form

$y-6=5(x-(-2)) \Rightarrow y-6=5(x+2)$

b) through $(2,-3)$ and parallel to $5x-3y=10$

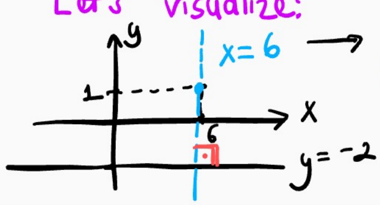
slope of $5x-3y=10$ is $m=\frac{5}{3}$ $\Bigg\}$ $\frac{3y}{3}=\frac{5x-10}{3} \Rightarrow y=\underbrace{\frac{5}{3}}_{m}x-\frac{10}{3}$

Parallel lines have the same slope.

The equation of the line that passes through $(2,-3)$ and has a slope of $m=\frac{5}{3}$ is: $\boxed{y-(-3)=\frac{5}{3}(x-2)}$

c) line through $(6,1)$ and perpendicular to $y=-2$

Let's visualize:



$\rightarrow$ $\boxed{x=6}$