Note Title 1/27/2020

Lecture #2

RURT (5% of class grade) → Recitation #2

Chapter 2 – Limits and Continuity

2.1- Introduction to Limits

The limit of a function is a tool to investigat the behavior of the function ($f(x)$) as x approaches to a certain number c.

Exp) Distance of a free-falling object is $s(t) = 16t^2$ ft. Find the average velocity of this object between $t_1 = 1$ s. to $t_2 = 2$ s.

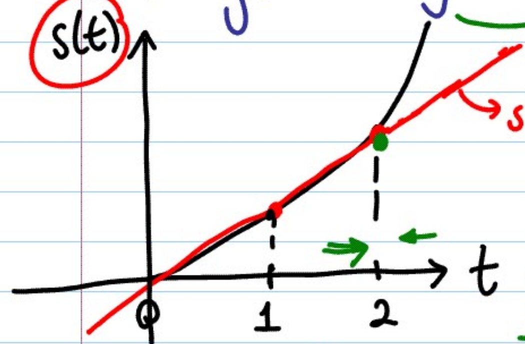
Solution: $\underbrace{\text{average velocity}}\ (\bar{v}) = \dfrac{\Delta s}{\Delta t}\ \dfrac{(\text{distance traveled})}{(\text{elapsed time})}$

$$\bar{v} = \frac{s(t_2) - s(t_1)}{t_2 - t_1} \qquad \frac{\Delta s}{\Delta t} \to \frac{ft.}{sec.}$$

$$= \frac{16 \cdot 2^2 - 16 \cdot 1^2}{2-1} = \frac{16 \cdot 4 - 16}{1} = 64 - 16 = \boxed{48\ \frac{ft}{sec.}}$$

How do we estimate the velocity at $t = 2\,\text{sec}$?

Average velocity vs. instantenous velocity



$s(t) = 16t^2$

slope of secant line $= \bar{v}$

time interval	Δt (interval length)	ave velocity
[1.99, 2]	0.01	63.84
[1.999, 2]	0.001	63.98
[2, 2.001]	0.001	64.016
[2, 2.01]	0.01	64.16

$$\bar{v} = \frac{s(2) - s(1.99)}{2 - 1.99}$$

$\Delta t \to 0$, $\bar{v} \to 64\,\text{ft/sec.}$

How can we calculate $v(2)$ exactly?

Let's assume a very tiny # h $(h \to 0)$

Instead of $[2, 2.00001]$ let's use

$$[2, 2+h]: \quad \bar{v} = \frac{s(2+h) - s(2)}{(2+h) - 2}$$

$s(t) = 16t^2$

$s(2+h) = 16(2+h)^2$

$s(2) = 16 \cdot 2^2$

ave. velocity

$$\bar{v} = \frac{16(2+h)^2 - 16\cdot 2^2}{\cancel{2}+h - \cancel{2}} = \frac{16(4+4h+h^2) - 16\cdot 4}{h}$$

$$= \frac{\cancel{16\cdot 4} + 16\cdot 4h + 16\cdot h^2 - \cancel{16\cdot 4}}{h} = \frac{\cancel{h}(64+16h)}{\cancel{h}} = 64 + \underbrace{16h}_{h \to 0}$$

$\boxed{\bar{v} = 64}$

Informal Definition of Limits

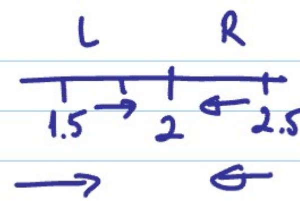
$\lim_{x \to c} f(x) = L$ "The limit of f(x) as x approaches c is L"

f(x) becomes arbitrarily close (virtually the same) to L by choosing x sufficiently close to c
(not equal to c)

$\lim_{h \to 0} \bar{v} = 64$; $\lim_{\Delta t \to 0} \bar{v} = 64$; $\lim_{t \to 2} \frac{s(t)-s(2)}{t-2} = 64$

Exp) Use a table to estimate

$$\lim_{\underbrace{x \to 2}_{x \neq 2}} \frac{x^2-x-2}{x-2}$$



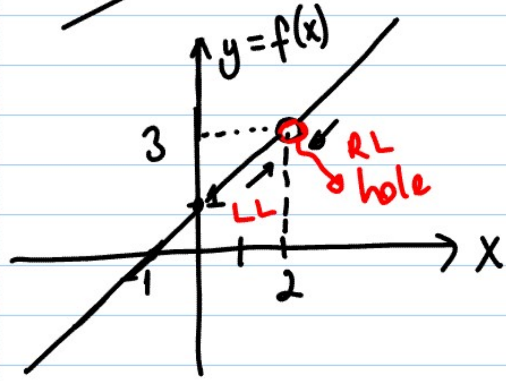
x	$y = f(x) = \frac{x^2-x-2}{x-2}$
1.5	2.5
1.9	2.9
1.99	2.99
2	undefined
2.01	3.01
2.1	3.1
2.5	3.5

$x \to 2$ $y \to 3$

$$f(x) = \frac{x^2-x-2}{x-2} \begin{cases} \text{if } x=2 & \text{undefined} \\ \text{if } x \neq 2 & \dfrac{\cancel{(x-2)}(x+1)}{\cancel{x-2}} = x+1 \end{cases}$$

$m=1,\ b=1$

~~$f(x) = x+1$~~



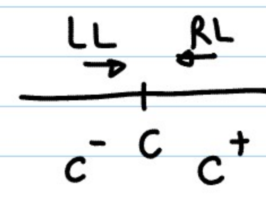
$$\lim_{x \to 2} f(x) = 3$$

One-sided limits

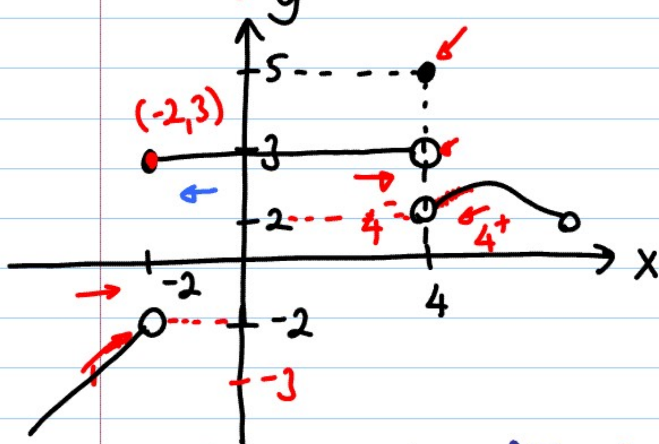
The <u>two-sided limit</u> $\lim_{x \to c} f(x)$ exists;

if and only if (IFF) the two-sided limits (right limit and the left limit) exist and they're equal.

$$\underbrace{\lim_{x \to c^-} f(x)}_{\text{Left-limit}} = L = \underbrace{\lim_{x \to c^+} f(x)}_{\text{Right-limit}}$$



Exp) Inspect the limits



$f(-2) = 3$

$\lim_{x \to -2^-} f(x) = -2$ → LL

$\lim_{x \to -2^+} f(x) = 3$ RL

$\lim_{x \to -2} f(x) \Rightarrow$ Does Not Exist (DNE)

$(-2 \neq 3)$

$\lim_{x \to 4^-} f(x) = 3$ LL

$\lim_{x \to 4^+} f(x) = 2$ RL

$\lim_{x \to 4} f(x)$ DNE two-sided limit

$(2 \neq 3)$

$f(4) = 5$

$(4,5)$

Limits that do not exist

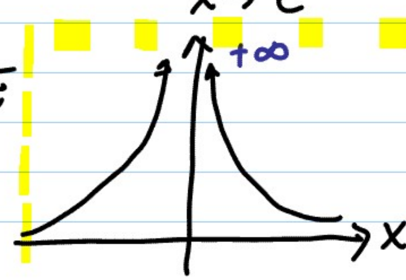
① one-sided limits not equal

If $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$ The $\lim_{x \to c} f(x)$ DNE

② Infinite limits (will cover in Ch. 4)

If $\lim_{x \to c} f(x) = \infty$ or $\lim_{x \to c} f(x) = -\infty$

The $\lim_{x \to c} f(x)$ DNE

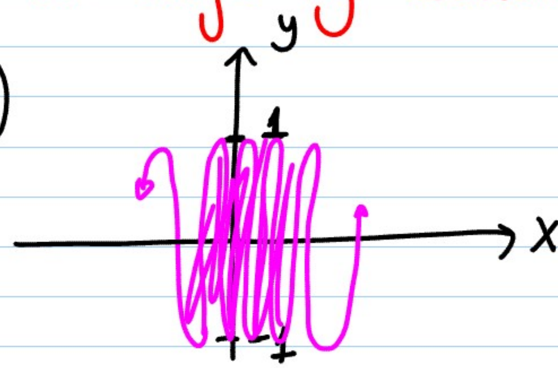


$f(x) = \frac{1}{x^2}$

$\lim_{x \to 0} \left(\frac{1}{x^2}\right)$ DNE

(3) A function that diverges by oscillation

$$\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$$



You try it!

$$\lim_{x \to 0} \frac{1 - \frac{1}{x+1}}{x} = ?$$

Hint:
try w/ Algebra