Note Title 3.8 Continued -- marginal Analysis 3/5/2020

Review: $x \to$ # of items

$p(x) \to$ price per item

$R(x) = p(x) \cdot x$ (revenue f.) $C(x)$ (cost f.)

Algebra exact/actual $C(x), R(x) \to \Delta C, \Delta R$

Calculus marginal analysis/estimate $C'(x), R'(x)$

Be careful! Estimate the cost of producing $(x+1)^{th}$ unit:

$C'(x)$

Estimate the revenue from producing $(x+1)^{th}$ unit:

$R'(x)$

Exp) $C(q) = 3q^2 + q + 500$ [$q \to$ # of units produced]

a) compute the actual cost of producing the 41^{th} unit.

{cost of producing 41 units minus 40 units}

$\Delta C = C(41) - C(40)$

$(3\cdot 41^2 + 41 + 500) - (3\cdot 40^2 + 40 + 500)$

$\underbrace{3\cdot 41^2} + \cancel{41} + \cancel{500} - \underbrace{3\cdot 40^2} \; 1 \; - \cancel{40} - \cancel{500}$

$3(41^2 - 40^2) + 1$

$3(41-40)(41+40)+1$

$3\cdot 1\cdot 81 + 1 = 243 + 1 = \underbrace{\$244}_{\text{actual cost}}$

b) Use marginal analysis to estimate the cost of producing the $\underbrace{41^{th}}$ unit.

$C'(40)$

$C(q) = 3q^2 + q + 500 \Rightarrow C'(q) = 6q + 1$

$C'(40) = 6\cdot 40 + 1 = \underbrace{\$241}_{\text{estimated cost}}$

Error Propagation

How an error in measuring a variable etc. affects the calculation of another variable.
(E.g: how Δr affects ΔV)

Exp) Side of a cube is measured as 10 cm. long. From this, volume of the cube is found to be $10^3 = 1000$ cm^3. If the original measurement of the side is accurate within 2%, approximately how accurate is the calculation of volume?

Recall: In differentials: $y = f(x)$

$$\frac{dy}{dx} = f'(x)$$

$$\boxed{dy = f'(x) \cdot dx}$$

$f(x) \to$ function of interest $(V(x))$

$\underbrace{\Delta V}_{\text{err. in calculating Vol.}} \approx V'(x) \cdot \underbrace{\Delta x}_{\text{err. in side measurement}}$ ↳ side

$x = 10$ cm.

measurement of x is off by $\pm 2\%$

$\Delta x = (\pm 2\%) \cdot 10 = \mp 0.2$ cm.

$V(x) = x^3 \Rightarrow V'(x) = 3x^2$

$V'(10) = 3 \cdot 10^2 = 300$

$\Delta V \approx V'(10) \cdot \Delta x$

$\Delta V \approx 300 \cdot (\mp 0.2) = \pm 60 \text{ cm}^3$

$\% \text{ error} = \frac{6\cancel{0}}{100\cancel{0}} = 6\%$

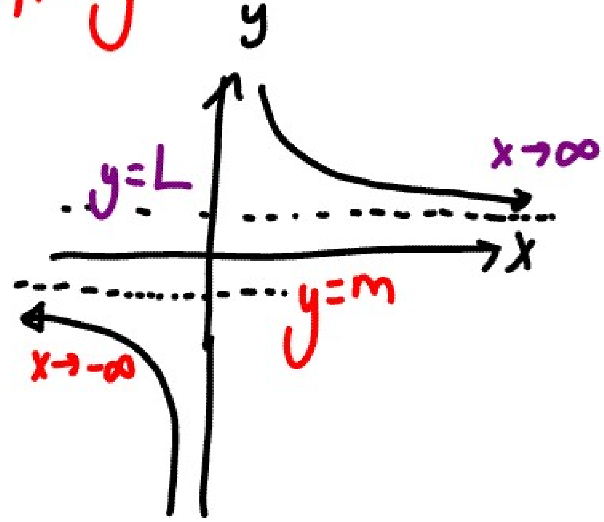
max. error $= |\Delta V| = |\pm 60| = 60 \text{ cm}^3$

4.4. Limits Involving Infinity

Limits at Infinity

$$\lim_{x \to \infty} f(x) = L$$

$$\lim_{x \to -\infty} f(x) = m$$



Special Limits:

A is any real #, r is a positive rational #

$$\lim_{x \to \infty} \frac{A}{x^r} = 0, \quad \lim_{x \to -\infty} \frac{A}{x^r} = 0$$

E.g.: $\lim_{x \to \infty} \frac{-3}{x^5} = 0$, $\lim_{x \to \infty} \frac{5}{\sqrt{x}} = 0$

Limits of polynomials at ∞

p(x), q(x) are polynomials

$$\lim_{x \to \mp\infty} \frac{p(x)}{q(x)}$$

PROCEDURE

[factor out the highest power of x in the denominator]

Exp) Calculate $\lim_{x \to \infty} \frac{3x^3-5x+9}{5x^3+2x^2-7}$ factor out $\to x^3$

$$\lim_{x \to \infty} \frac{x^3\left(3 - \cancel{\frac{5}{x^2}}^{\,0} + \cancel{\frac{9}{x^3}}^{\,0}\right)}{x^3\left(5 + \cancel{\frac{2}{x^1}}^{\,0} - \cancel{\frac{7}{x^3}}^{\,0}\right)}$$

use special limits at ∞

$$\boxed{\lim_{x \to \mp\infty} \frac{A}{x^r} = 0}$$

$$\lim_{x \to \infty} \left(\frac{\cancel{x^3}(3-0+0)}{\cancel{x^3}(5+0-0)}\right) = \frac{3}{5}$$

Exp) Calculate $\lim_{x \to -\infty} \left(\frac{5x^3-2x^2+1}{2x^2-4}\right)$ $\to$ factor out x^2

$$\lim_{x \to -\infty} \frac{\cancel{x^2}\left(5x-2+\cancel{\frac{1}{x^2}}^{\,0}\right)}{\cancel{x^2}\left(2-\cancel{\frac{4}{x^2}}^{\,0}\right)}$$

$$= \lim_{x \to -\infty} \frac{(5x-2)}{2} \overset{\text{"OSP"}}{=} \frac{5(-\infty)-2}{2} = \frac{-\infty}{2} = \boxed{-\infty}$$

Exp) $\lim_{x \to -\infty} \left(\frac{9x^3+5x+30}{x^5-1} \right) \to$ factor out x^5

$$\lim_{x \to -\infty} \left(\frac{\cancel{x^5}\left(\cancel{\frac{9}{x^2}}^{0} + \cancel{\frac{5}{x^4}}^{0} + \cancel{\frac{30}{x^5}}^{0}\right)}{\cancel{x^5}\left(1 - \cancel{\frac{1}{x^5}}^{0}\right)} \right) = \frac{0}{1} = 0$$

Exp) Calculate $\lim_{x \to -\infty} \left(\frac{\sqrt{3x^2-6}}{5+2x} \right) \to$ factor out x

$$\lim_{x \to -\infty} \left(\frac{\sqrt{x^2\left(3 - \frac{6}{x^2}\right)}}{x\left(\frac{5}{x}+2\right)} \right)$$

$$\sqrt{x^2} = |x| \begin{cases} x \geq 0, & x \\ x < 0, & -x \end{cases}$$

$$|-2| = -(-2) = 2$$

$$\lim_{x\to -\infty}\left(\frac{|x|\cdot\sqrt{3-\frac{6}{x^2}}}{x\left(\frac{5}{x}+2\right)}\right)=\lim_{x\to -\infty}\left(\frac{\cancel{-x}\cdot\sqrt{3-\cancel{\frac{6}{x^2}}\to 0}}{\cancel{x}\left(\cancel{\frac{5}{x}}\to 0+2\right)}\right)$$

$$=\frac{-\sqrt{3}}{2}$$

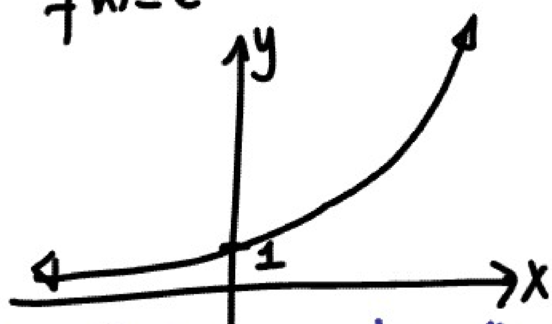
Exp) Calculate $\lim_{x\to\infty}\frac{\sqrt{3x^2-6}}{5+2x}$

$$\lim_{x\to\infty}\left(\frac{|x|\sqrt{3-\frac{6}{x^2}}}{x\left(\frac{5}{x}+2\right)}\right)=\lim_{x\to\infty}\left(\frac{\cancel{x}\cdot\sqrt{3-\cancel{\frac{6}{x^2}}\to 0}}{\cancel{x}\left(\cancel{\frac{5}{x}}\to 0+2\right)}\right)$$

$$=\frac{\sqrt{3}}{2}$$

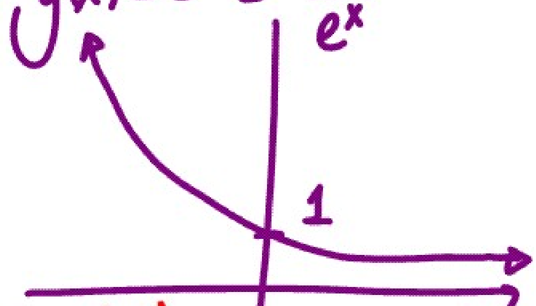
Limits Involving e^x

$f(x) = e^x$



$\lim_{x \to -\infty} e^x = 0$ $\qquad$ $\lim_{x \to \infty} e^x = +\infty$

$g(x) = e^{-x} = \frac{1}{e^x}$



$\lim_{x \to -\infty} (e^{-x}) = \infty$ $\qquad$ $\underbrace{\lim_{x \to \infty} (e^{-x}) = 0}$

Exp) Calculate $\lim_{x \to -\infty} \frac{e^x - 1}{4 + 5e^x}$ $\qquad$ $\underbrace{\left[\lim_{x \to -\infty} e^x = 0\right]}$

$$\lim_{x \to -\infty} \left(\frac{e^{x} - 1}{4 + 5 \cdot e^{x}} \right) = \frac{-1}{4}$$

Exp) Calculate $\lim_{x \to \infty} \left(\frac{e^x - 1}{4 + 5e^x} \right)$ $\qquad$ $\left[\lim_{x \to \infty} (e^x) = \infty\right]$

$\cancel{\text{"DSI"} = \frac{\infty - 1}{4 + 5 \cdot \infty} = \text{"}\frac{\infty}{\infty}\text{"} = 1}$ $\quad$ undefined / Indeterminate form

how NOT to do it!

How to do it ↓

"factor out e^x"

$$\lim_{x\to\infty}\left(\frac{\cancel{e^x}\left(1-\frac{1}{e^x}\right)}{\cancel{e^x}\left(\frac{4}{e^x}+5\right)}\right)$$

$$\lim_{x\to\infty}\left(\frac{1-\cancel{\frac{1}{e^x}}^{\to 0}}{\cancel{\frac{4}{e^x}}^{\to 0}+5}\right)=\frac{1}{5}$$

Recall:

$$\left[\lim_{x\to\infty}\left(\frac{1}{e^x}\right)=0\right]$$

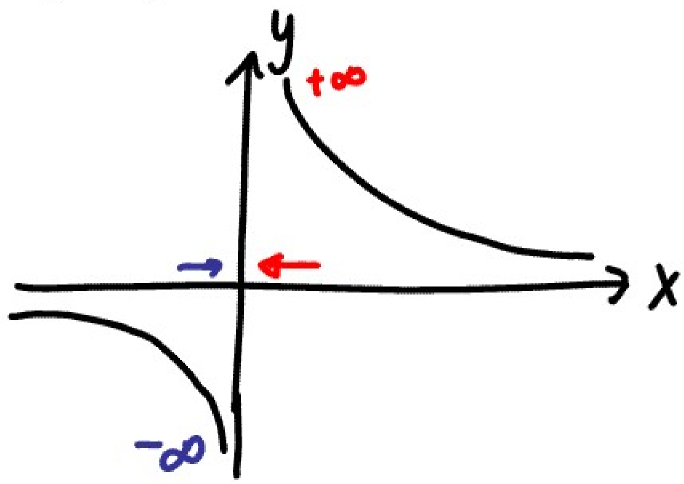
Infinite Limits

$\lim_{x \to c} f(x) = {}^{+}\infty$ or $\lim_{x \to c} f(x) = {}^{-}\infty$

f increases without a bound as x approaches to a constant number (c)

f decreases w/out bound as x approaches to a constant # (c)

E.g: $\lim_{x \to 0^+} \left(\frac{1}{x}\right) = +\infty$, $\lim_{x \to 0^-} \left(\frac{1}{x}\right) = -\infty$



Procedure for Finding Infinite Limits

Try "DSP". If you obtain "$\frac{\text{nonzero \#}}{0}$",

then each one-sided limit is ∞.

Do a sign analysis for numerator, denominator to determine the sign of ∞ as $-\infty$ or ∞.

Exp) Calculate $\lim\limits_{x \to 2^-} \left(\frac{3x-5}{x-2} \right)$

"$\frac{\text{non-zero \#}}{0}$"

$$\lim_{x \to 2^-} \left(\frac{3x-5}{x-2} \right) \overset{\text{"DSP"}}{=} \frac{3 \cdot 2 - 5}{2^- - 2} = \frac{1}{0^-} = -\infty$$

↳ (-) number very close to 0

Exp) Calculate $\lim_{x \to 2^+} \left(\frac{3x-5}{x-2} \right)$

$$\frac{\text{"non-zero \#"}}{0}$$

$$\lim_{x \to 2^+} \left(\frac{3x-5}{x-2} \right) \overset{\text{"DSP"}}{=} \frac{3 \cdot 2 - 5}{2^+ - 2} = \frac{1}{0^+} = +\infty$$

↳ (+) # very close to 0.