Supplementary Problem - 4.5.

Q12 Page 280 - "B Level"

$$\lim_{x\to 0} \frac{x-\sin x}{x\cdot\tan x} \overset{\text{"DSP"}}{=} \frac{0-\sin 0}{0\cdot\tan 0} = \frac{0}{0\cdot 0} = \text{"}\frac{0}{0}\text{"}$$

Use L.R.

$$\lim_{x\to 0} \frac{1-\cos x}{1\cdot\tan x + x\cdot\sec^2 x} \overset{\text{"DSP"}}{=} \frac{1-\cos 0}{\tan 0 + 0\cdot\sec^2 0} = \text{"}\frac{0}{0}\text{"}$$

(product rule for x·tanx)

Use L.R. again

$$\lim_{x\to 0} \frac{\sin x}{\sec^2 x + 1\cdot\sec^2 x + x\cdot 2\cdot\sec x\cdot\tan x\cdot\sec x} \overset{\text{"DSP"}}{=} \frac{0}{1+1+0} = \boxed{0}$$

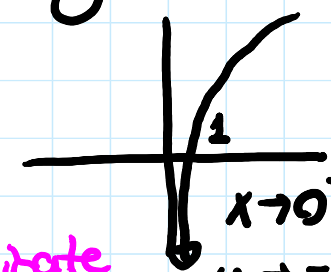
(product rule for $x\cdot\sec^2 x$)

Q37 – Page 281 "A Level"

$$\lim_{x\to 0^+}(e^x-1)^{\frac{1}{\ln x}} \overset{\text{"OSP"}}{=} (e^0-1)^{\frac{1}{\ln 0^+}} \overset{\text{"OSP"}}{=} (1-1)^{\frac{1}{-\infty}} = \underbrace{0^0}_{\text{"Other" Indeterminate Form}}$$

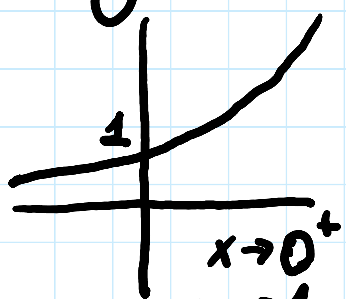
↓ Recall ↓

Graph of $y=\ln x$



$x\to 0^+$
$y\to -\infty$

Graph of $y=e^x$



$x\to 0^+$
$y\to 1$

Use Algebra to re-write:

Let $L=\lim_{x\to 0^+}(e^x-1)^{\frac{1}{\ln x}}$

$$\ln L=\ln\left(\lim_{x\to 0^+}(e^x-1)^{\frac{1}{\ln x}}\right)$$

$$\ln L=\lim_{x\to 0^+}\left(\ln (e^x-1)^{\frac{1}{\ln x}}\right)$$

$$\boxed{\ln L}=\lim_{x\to 0^+}\left(\frac{1}{\ln x}\cdot \ln(e^x-1)\right)=\lim_{x\to 0^+}\left(\frac{\ln(e^x-1)}{\ln x}\right)$$

"Use OSP"

$$\overset{\text{"OSP"}}{=}\frac{\ln(e^{0^+}-1)}{\ln 0^+}=\frac{\ln(1^+-1)}{\ln 0^+}=\frac{\ln 0^+}{\ln 0^+}=\frac{-\infty}{-\infty}\quad \text{"Indeterminate form"}$$

"Use LR"

$$=\lim_{x\to 0^+}\frac{\frac{e^x}{e^x-1}}{\frac{1}{x}}=\lim_{x\to 0^+}\frac{e^x\cdot x}{e^x-1}\overset{\text{"OSP"}}{=}\text{"}\frac{0}{0}\text{"}$$

"Use LR again"

$$=\lim_{x\to 0^+}\frac{e^x\cdot 1+e^x\cdot x}{e^x}=\lim_{x\to 0^+}\frac{\cancel{e^x}(1+x)}{\cancel{e^x}}=\boxed{1}$$

Final Answer

$\ln L=1$

$\boxed{L=e}$