Math 135, Worksheet 4.3 Solutions, Spring 2020

1. a. Consider the function f and its derivatives below.

$$f(x) = \frac{2x^2 - 3x}{x-2}, f'(x) = \frac{2(x-3)(x-1)}{(x-2)^2}, f''(x) = \frac{4}{(x-2)^3}$$

Find the vertical and horizontal asymptotes of f. Then find where f is decreasing, where f is increasing, where f is concave down, and where f is concave up. Calculate the x-coordinates of all local minima, local maxima, and points of inflection. Write "NONE" for your answer if appropriate.Intervals should be given in a comma-separated list and should be as inclusive as possible. Sketch the graph on the grid provided. Fill in the table below. ONLY THE ANSWERS IN THE TABLE AND GRAPH WILL BE GRADED.

vertical asymptote(s):	$x = 2$
horizontal asymptote(s):	NONE
where f is decreasing:	$(1, 2), (2, 3)$
where f is increasing:	$(-\infty, 1), (3, \infty)$
x-coordinate(s) of local minima:	$x = 3$
x-coordinate(s) of local maxima:	$x = 1$
where f is concave down:	$(-\infty, 2)$
where f is concave up:	$(2, \infty)$
x-coordinate(s) of inflection point(s):	NONE

Solution:

- Vertical asymptote(s):

  $f(x)$ is continuous for all real numbers except $x = 2$. Therefore, our candidate vertical asymptote is the line $x = 2$. Direct substitution of $x = 2$ into $f(x)$ results in a zero in the denominator, which indicates that both one-sided limits are infinite. Now, lets find the left- and right- hand limits of $f(x)$ as x approaches to 2.

$$\lim_{x \to 2^-} f(x) = -\infty$$

$$\lim_{x \to 2^+} f(x) = \infty$$

  Therefore, the line $x = 2$ is a true vertical asymptote.

- Horizontal asymptote(s):

  Use L'Hôpital's Rule, to compute the limits at $\pm\infty$.

$$\lim_{x\to\pm\infty}\left(\frac{2x^2-3x}{x-2}\right)=\lim_{x\to\pm\infty}\left(\frac{2x-3}{1-\frac{2}{x}}\right)=\pm\infty$$

  Therefore, there are no horizontal asymptotes.

- intervals of decrease/increase:

  The first-order critical numbers of $f(x)$ are those x-values such that: $f'(x)=0$ or does not exist. Therefore, the critical numbers are $x=3$ and $x=1$. When we construct a sign chart for $f'(x)$ by using the critical numbers and the x-coordinate of the vertical asymptote ($x=2$) we see that $f(x)$ is decreasing on the intervals of $(1,2)$ and $(2,3)$; and increasing on the intervals of $(-\infty,1)$ and $(3,\infty)$.

- x-coordinate(s) of local minima:

  Based on the sign chart for $f'(x)$, since f transitions from decreasing to increasing at $x=3$, there is a local minima at $x=3$.

- x-coordinate(s) of local maxima:

  Based on the sign chart for $f'(x)$, since f transitions from increasing to decreasing at $x=1$, there is a local maxima at $x=1$.

- intervals of concavity:

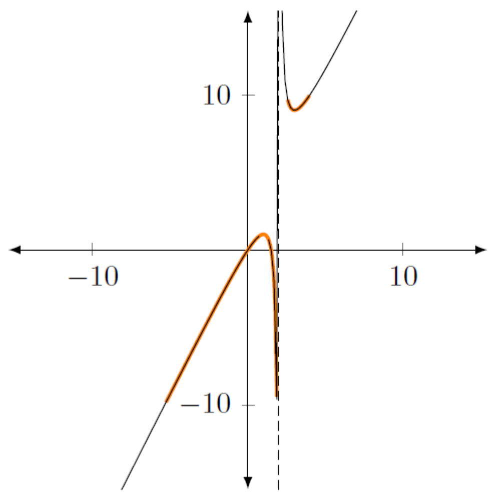
  The second-order critical numbers of f(x) are those x-values such that: $f''(x)=0$ or does not exist. $f''(x)$ is never equal to 0. However, since the x-coordinate of the vertical asymptote is $x=2$, we should include $x=2$ as a cutpoint of the sign chart for $f''(x)$. Based on the sign chart for $f''(x)$, f transitions from concave down to concave up at $x=2$.

- x-coordinate(s) of inflection point(s):

  Although there is a change in concavity, $x=2$ is not the point of inflection since the function is not continuous at the x-coordinate of the vertical asymptote. Therefore, there is no point of inflection.

2. b. Sketch the graph of $f(x)$ on the provided grid below below. Make sure to label the scales on the axes! For each local extremum or inflection point, identify its coordinates and label the point "local min.", "local max", or "infl. pt." as appropriate.

Solution:



$$f(x) = \frac{2x^2 - 3x}{x - 2}$$