Identify the Error

Wednesday, October 14, 2020 9:17 PM

Identify and correct the error in the following argument. Suppose $y^2 + 2y = 2x^3 - 7$. Differentiating both sides with respect to x to find $\frac{dy}{dx}$, we have $2y + 2\frac{dy}{dx} = 6x^2$, which implies that $\frac{dy}{dx} = 3x^2 - y$.

The error was: the student did not use implicit differentiation correctly.
The error was corrected below:

$$2y \cdot \frac{dy}{dx} + 2 \cdot \frac{dy}{dx} = 6x^2$$

$$\frac{\frac{dy}{dx}(2y+2)}{2y+2} = \frac{6x^2}{2y+2}$$

$$\frac{dy}{dx} = \frac{\cancel{6}x^2}{\cancel{2}(y+1)} = \frac{3x^2}{y+1} \checkmark$$

3.11 Related Rates

Monday, October 19, 2020 8:24 AM

We will relate the rates of change of different variables with respect to time.

Procedure for solving related rates prb.

1. Draw a figure, assign variables to quantities that vary (What's not changing (a constant) vs. what's changing (variable))

2. Find a formula or an equation that relates the variables

3. Differentiate the equation (usually implicitly w/ respect to time)

4. Substitute specific values and solve algebraically for any required rate (use correct units)

Steps 1 & 2 involve reading & interpreting the prb.
Step 3 is implicit differentiation / Step 4 Algebra

Exp1)

Monday, October 19, 2020 3:43 PM

Exp1) A spherical balloon is filled w/ gas. When $r = 2ft$, the radius is increasing at the rate of $\frac{1}{6} \frac{ft}{min}$. How fast is the volume changing at this time?

$$\left(V = \frac{4}{3}\pi r^3\right)$$

Solution:

① air

balloon

$r \to$ radius

$V \to$ volume

Given: $r = 2ft.$

$$\frac{dr}{dt} = +\frac{1}{6}\,\frac{ft}{min}$$ (rate of change of radius wrt time)

Asked: $$\frac{dV}{dt} = ?$$ (how is the Volume changing wrt time?)

② Formula: $V = \frac{4}{3}\pi r^3$

③ Implicit Diff. wrt (w/respect to) time:

$$(V = V(t)) \quad V = \underbrace{\frac{4}{3}\pi}_{\text{constant}} r^3 \quad (r = r(t))$$

$$\frac{dV}{dt} = \frac{4}{3} \cdot \pi \cdot 3 \cdot r^2 \cdot \frac{dr}{dt}$$

Given:

(4) At the moment when $r = 2ft$, $\frac{dr}{dt} = \frac{1}{6}$ ft/min.

substitute these speficic values in the eq. step (3)

$$\frac{dV}{dt} = \frac{4}{3} \cdot \pi \cdot 3 \cdot r^2 \cdot \frac{dr}{dt}$$

$$\frac{dV}{dt} = \frac{\cancel{4}^{2}}{\cancel{3}} \cdot \pi \cdot \cancel{3} \cdot 2^2 \cdot \frac{1}{\cancel{6}_{3}} = \frac{8\pi}{3} \frac{ft^3}{min.}$$

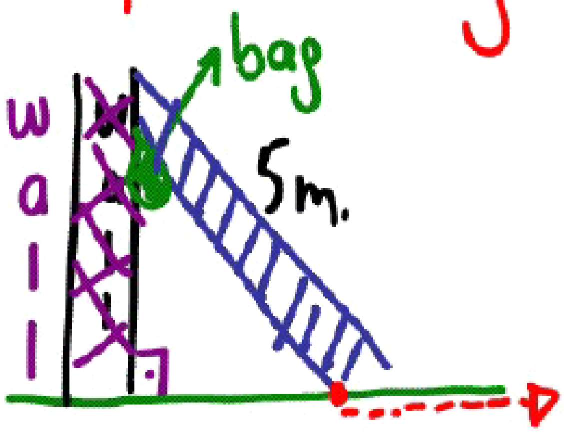
Volume is increasing at a rate of $\frac{8\pi}{3} \frac{ft^3}{min}$. *

Units* how do we measure volume? in this
how do we measure time? prb.

Exp2

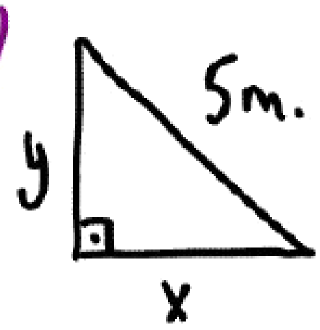
Monday, October 19, 2020 3:46 PM

Exp2) Leaning Ladder Prb.



when the foot of the ladder is 4m away from the wall and the foot is moving away at the rate of 2 m/sec. how fast is the bag descending?

(1)



Given:

x → distance between foot of the ladder and the wall

y → distance between the bag and the ground

Asked: when $x=4m.$, $\frac{dx}{dt}=2m/s.$ $\frac{dy}{dt}=?$

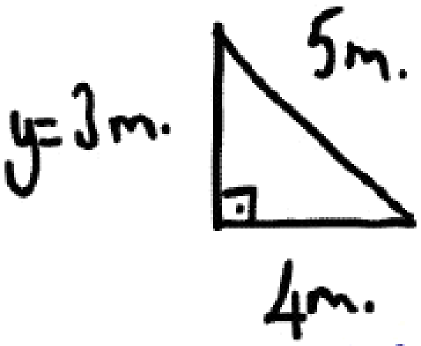
(Is y increasing wrt time? NO!)

(2) $$x^2+y^2=5^2$$

(3) $$2x\cdot\frac{dx}{dt}+2y\cdot\frac{dy}{dt}=0$$

Given: Asked:

(4) when $x = 4m.$, $\frac{dx}{dt} = 2m/s.$; $\frac{dy}{dt} = ?$



3-4-5 Special Right Triangle

$y^2 + 4^2 = 5^2$

$y^2 = 25 - 16 = 9$

$y = 3m.$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$

Substitute given (specific) values:

$$2 \cdot 4 \cdot 2 + 2 \cdot 3 \cdot \frac{dy}{dt} = 0$$

$$16 + 6 \cdot \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-16}{6} = \frac{-8}{3} \ m/sec.$$

The bag is descending at a rate of $\frac{8}{3} \frac{m}{sec}$.

already implies a neg. change