3.7 The Chain Rule

Sunday, October 11, 2020 8:31 PM

3.7 The Chain Rule

The differentiation rules presented so far allow us to find derivatives of many functions. However, these rules are inadequate for finding the derivatives of most *composite functions*. Here is a typical situation. If $f(x) = x^3$ and g(x) = 5x + 4, then their composition is $f(g(x)) = (5x + 4)^3$. One way to find the derivative is by expanding $(5x + 4)^3$ and differentiating the resulting polynomial. Unfortunately, this strategy becomes prohibitive for functions such as $(5x + 4)^{100}$. We need a better approach.



g-nordef. f-) notside f.

$$f(x) = x^{3} \qquad (x^{3})' = 3 \cdot x^{2}$$

$$f(g(x)) = (5x+4)^{3} \qquad ((5x+4)^{3})'$$

$$(5x+4)^{100}$$

THEOREM 3.13 The Chain Rule

Suppose y = f(u) is differentiable at u = g(x) and u = g(x) is differentiable at x. The composite function y = f(g(x)) is differentiable at x, and its derivative can be expressed in two equivalent ways.

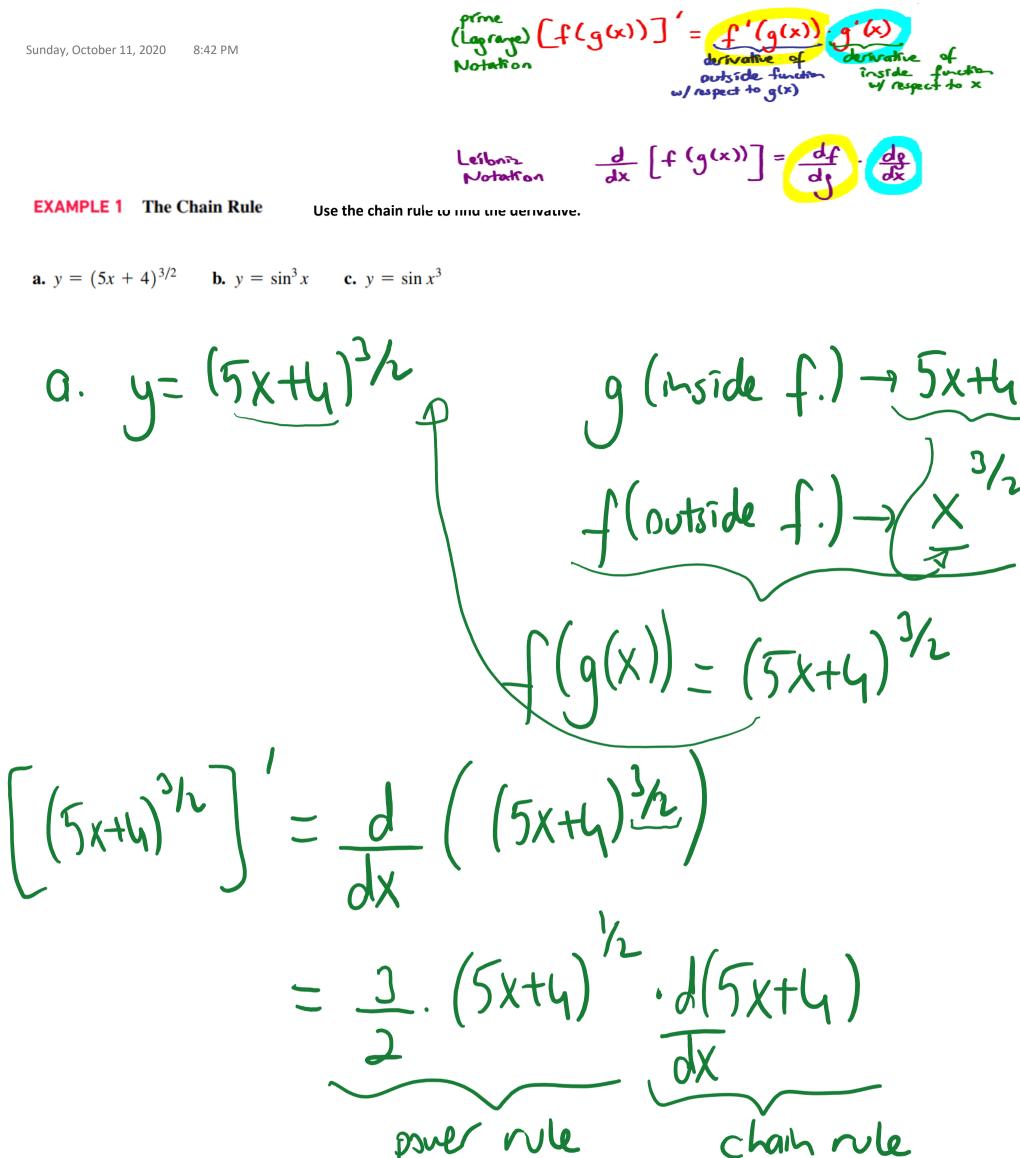
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \tag{1}$$

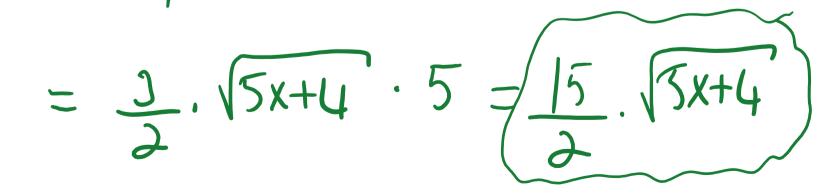
$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x) \tag{2}$$

prime
(Laprage)
$$\left(f(g(x))\right)' = f'(g(x)) g'(x)$$

Notation
Notation
Notation
Leibniz
Leibniz







Monday, October 12, 2020 10:40 AM Calculate the derivative: **a.** $y = (5x + 4)^{3/2}$ **b.** $y = \sin^3 x$ **c.** $y = \sin x^3 - 5i \ln (\chi^3)$ $g \rightarrow sm x$ (mother f.) g'(x) = cosx $f \rightarrow (x)$ (notside f.) $y = (SMx)^{3}$ $f(q(x)) = (Smx)^3$ $\frac{d}{dx}\left(\left(Shx\right)^{3}\right) = J\left(Shx\right)^{2} \cdot OSx$ C. $y = Sih(x^2)$

 $g \rightarrow x^{3}$ (inside f.) $f \rightarrow sin(x)$ (nutside f.) $f(g(x)) = sin(x^{3})$ $\left(\int_{M} (\chi^{j}) \right)' = \cos(\chi^{j}) \cdot \int_{X}$

 $(SM_X)^{S}$ VS. $SM_{X^{3}}$

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

THEOREM 3.14 Chain Rule for Powers

If g is differentiable for all x in its domain and p is a real number, then

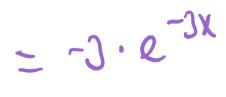
$$\frac{d}{dx}\left((g(x))^p\right) = p(g(x))^{p-1}g'(x).$$

EXAMPLE 2 The Chain Rule Use equation (2) of the Chain Rule to calculate the derivatives of the following functions.

a.
$$(6x^{3} + 3x + 1)^{10}$$
 b. $\sqrt{5x^{2} + 1}$ c. $(\frac{5x^{2}}{3x^{2} + 2})^{3}$ d. e^{-3x}
Q. $\frac{d}{dx} \left((\frac{6x^{3} + 3x + 1}{9})^{10} \right)^{2} = 40 \cdot (6x^{3} + 3x + 1)^{9} \cdot (6x^{3} + 3x + 1)^{9}$
 $= 10 \cdot (6x^{3} + 3x + 1)^{9} \cdot (18x^{2} + 3)$
b. $\frac{d}{dx} \left((5x^{2} + 1)^{1/2} \right) = \frac{1}{2} \cdot (5x^{2} + 1)^{-1/2} \cdot 10x$
evolut:
 $\frac{d}{dx} \left((5x^{2} + 1)^{1/2} \right) = 5x (5x^{2} + 1)^{-1/2} \cdot \frac{10x}{\sqrt{5x^{2} + 1}}$
C. $\frac{d}{dt} \left((\frac{5t^{2}}{3t^{2} + 2})^{2} \right) = 3 \cdot (\frac{5t^{2}}{3t^{2} + 2})^{2} \cdot \frac{d}{dt} (\frac{5t^{2}}{3t^{2} + 1})$
 $\frac{d}{dt} (\frac{5t^{2}}{3t^{2} + 2})^{2} = 3 \cdot (\frac{5t^{2}}{3t^{2} + 2})^{2} \cdot \frac{d}{dt} (\frac{5t^{2}}{3t^{2} + 1})$

 $= 3 \cdot \left(\frac{5t^{2}}{3t^{2}+2}\right)^{2} \cdot \left(\frac{10t \cdot (3t^{2}+2) - 5t^{2} \cdot 6t}{(3t^{2}+2)^{2}}\right)$ quotret rule result

39t3+29t-3)t3 Monday, October 12, 2020 10:56 AM $= 3 \cdot \left(\frac{5t^{2}}{3t^{2}+2} \right)^{2} \cdot \left(\frac{10t \cdot (3t^{2}+2) - 5t^{2} \cdot 6t}{(3t^{2}+2)^{2}} \right)^{2}$ quotret rule result 500 $3 \cdot 25t^4 \cdot 25t^4 \cdot 25t^4 \cdot 25t^4 \cdot 35t^4 \cdot$ $= 3 \cdot \left(\frac{5t^{1}}{3t^{2}+2} \right)^{2} \cdot \frac{20t}{(3t^{2}+2)^{2}} =$ $y=e^{-3x}$ $(e^{x}) = e^{x} \cdot 1$ d) p-]X' l Ł,g. $-e^{-3x} \cdot (-3)$ e-2×



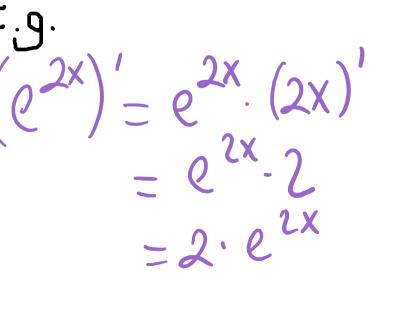


Table 3.3

x	f'(x)	g(x)	g'(x)
1	5	2	3
2	7	1	4

EXAMPLE 3 Calculating derivatives at a point Let h(x) = f(g(x)). Use the values in Table 3.3 to calculate h'(1) and h'(2).

$\frac{d}{dx} \left[f\left(g(x)\right) \right] = \frac{df}{dy} \cdot \frac{dg}{dx}$ Leibniz Notation

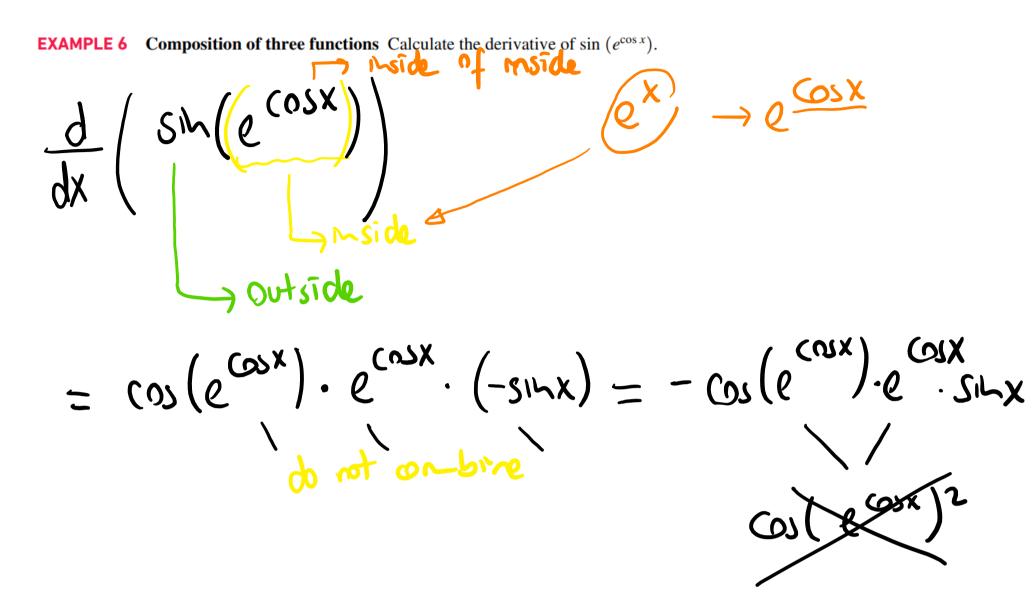
h(x) = f(g(x)) $(lagrage) \left(f(g(x)) \right)' = \left(f'(g(x)) \right) \left(g'(x) \right) \\ derivative of \\ outside function \\ w/respect to g(x) \\ w' respect to x \\ \end{pmatrix} \left((x) - f(g(x)) \right) \left((x) - f(g(x)) \right) \left((x) - f(g(x)) \right) \\ (x) - f(g(x)) - f(g(x)) \\ (x) -$ (ع=1)=2) (م'(1)=3) $h'(1) = \int (q(1)) \cdot q'(1)$ $= \int (2) \cdot 3$ 7.]=21

$h'(2) = f'(g(2)) \cdot g'(2) = -f'(1) \cdot g'(2) = 5 \cdot 4 = 20$

$$(e^{u})'=e^{u}\cdot u'$$

The Composition of Three or More Functions

We can differentiate the composition of three or more functions by applying the Chain Rule repeatedly, as shown in the following example.



EXAMPLE 7 Combining rules Find
$$\frac{d}{dx}(x^2\sqrt{x^2+1})$$
.
Pacall: $(f \cdot g)' = f \cdot g + f \cdot g'$ chain
 $\frac{d}{dx}(x^2 \cdot (x^2+1)^{l_{h}}) = 2x \cdot (x^2+1)^{l_{h}} + x^2 \cdot ((x^2+1)^{l_{h}})'$
 $= 2x \cdot (x^2+1)^{l_{h}} + x^2 \cdot \frac{1}{dx} \cdot (x^2+1)^{l_{h}}$
 $= \frac{2x \cdot \sqrt{x^2+1}}{(\frac{l}{(x^2+1)})} + \frac{x^2}{\sqrt{x^2+1}}$

$$= \frac{2x^{2}+2x}{\sqrt{x^{2}+1}+x^{3}} = \frac{3x^{3}+2x}{\sqrt{x^{2}+1}}$$