5.2 Definite Integrals, 5.3 FTC

Monday, January 18, 2021 2:46 PM

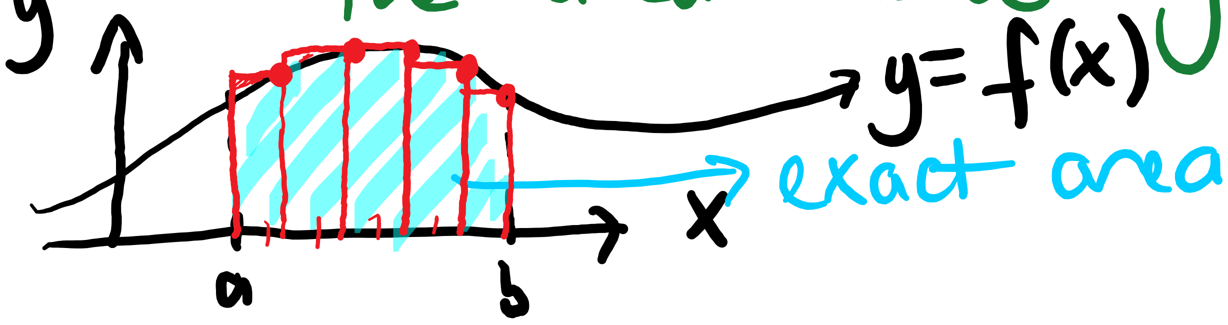
Keypoints

1) Review Riemann sum & geo. meaning
2) Define definite integral
3) Review Fundamental Theorem of Calculus (Part 1, Part 2)

Steps for Riemann Sum:

1) It's a method to approximate the area of a region bounded by $y=f(x)$ and the x-axis on $[a,b]$
2) Use rectangles with bases on the x-axis to ESTIMATE area
3) Divide $\underbrace{[a,b]}$ into n equal length sub-intervals that makes the basis of the n rect.
4) Choose the height of each rect. to be the function value at the right/left endpoints, midpoint

5) Total area of rectangles (R_n) ESTIMATES the area under graph.



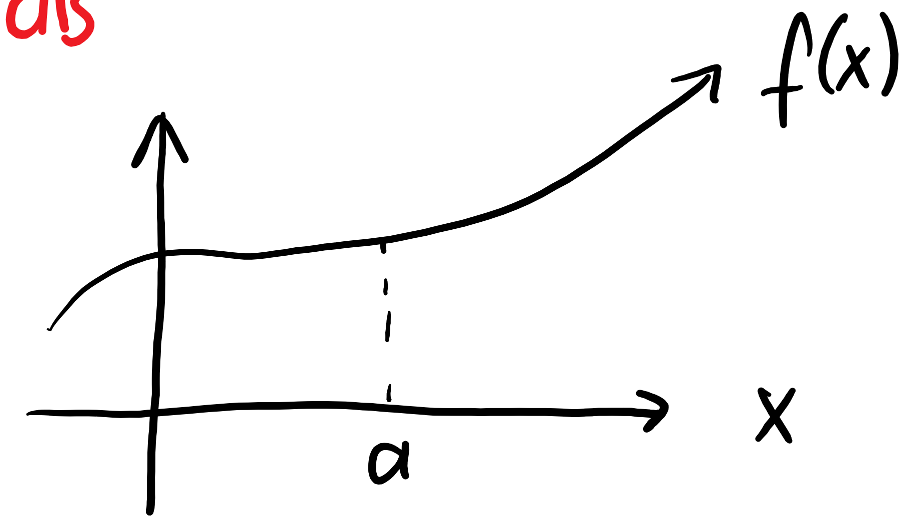
as $n \to \infty$

$$\lim_{n \to \infty} R_n \to \underbrace{A_{exact}}_{\underline{\text{def}}.\ \text{integral}}$$

$$\int_a^b f(x)\,dx$$

Properties of Integrals

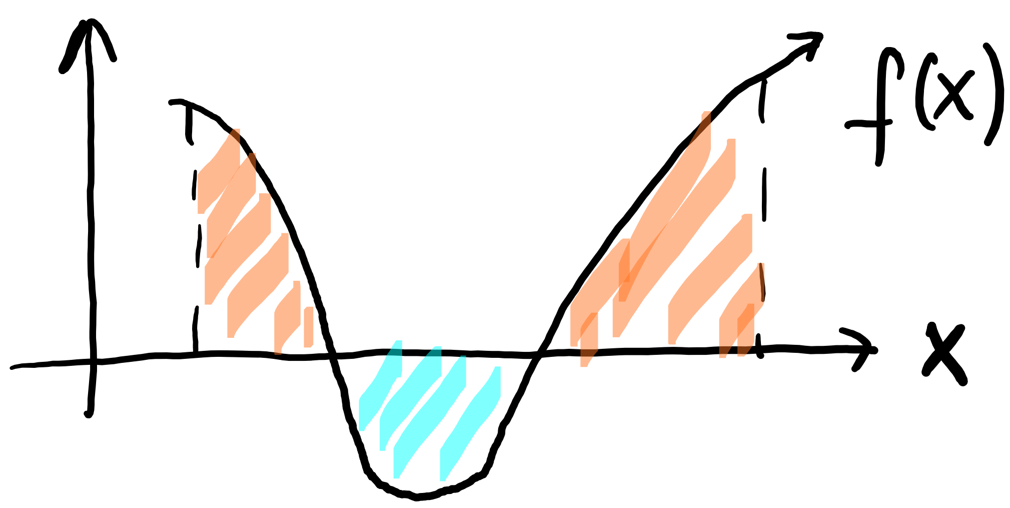
1) $\int_a^a f(x)\,dx = 0$



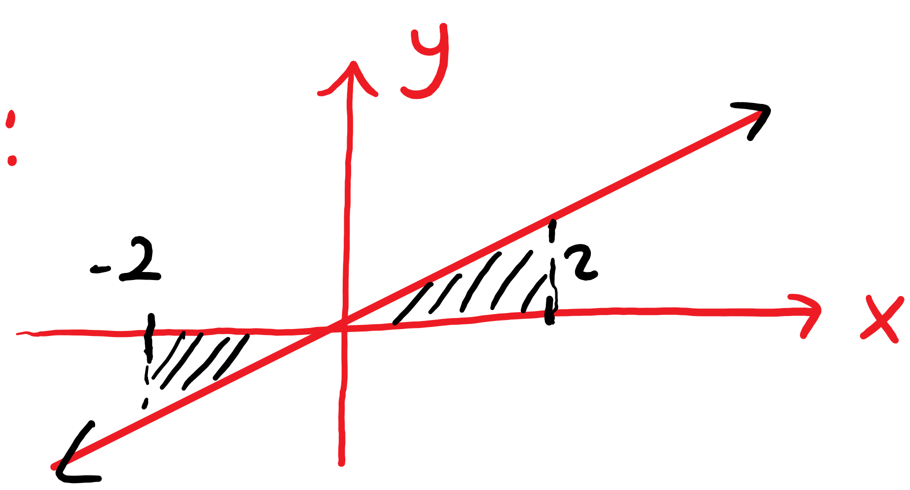
line segment has an area of 0

2) Net Area

$$\int_a^b f(x)\,dx = \left(\text{area above x-axis}\right) - \left(\text{area below x-axis}\right)$$



e.g.:



$f(x) = x$

$$\int_{-2}^{2} x \cdot dx = 0$$

$$\int_{-2}^{2} x^1 \cdot dx = \left.\frac{x^2}{2}\right|_{-2}^{2} = \frac{2^2}{2} - \frac{(-2)^2}{2} = \cancel{\frac{4}{2}} - \cancel{\frac{4}{2}} = 0$$

3) $$\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

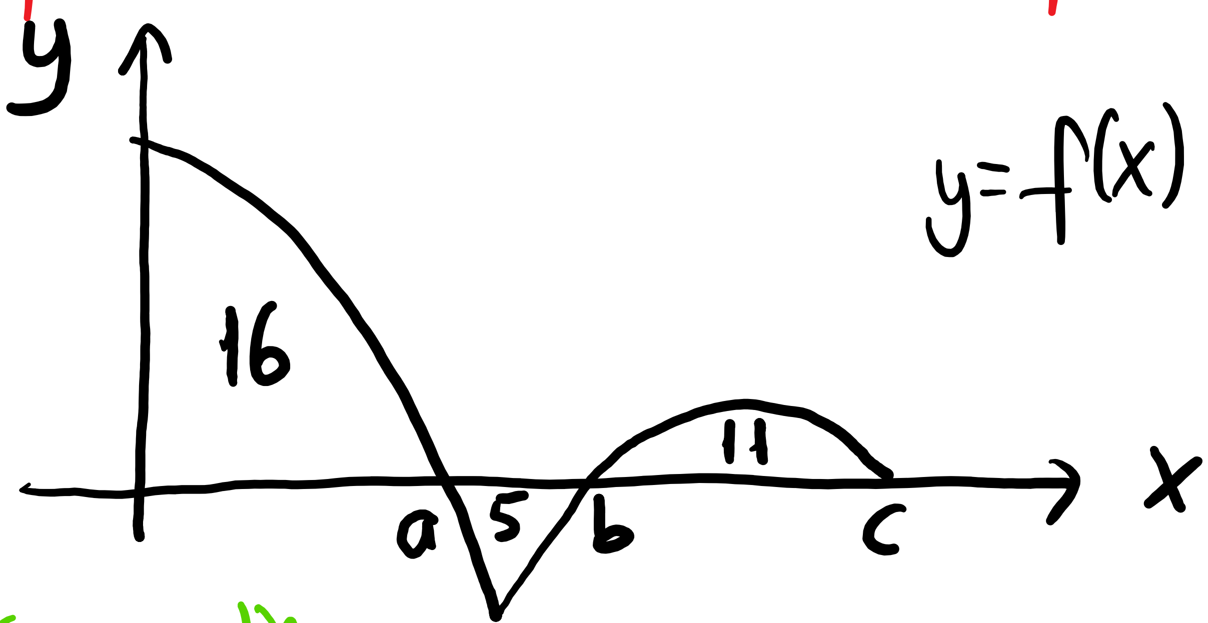
4) $$\int_a^b c \cdot f(x)\,dx = c \cdot \int_a^b f(x)\,dx$$

5) $$\int_a^b f(x)\,dx = \underbrace{\int_a^c f(x)\,dx} + \underbrace{\int_c^b f(x)\,dx}$$

6) $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

Exp1) Evaluate the following integrals

$f(x)$ vs $|f(x)|$



$c > a$

$\boxed{\text{a) } 6}$
b) 22
c) −6

upper limit of integration → c

a) $\int_a^c f(x)\,dx = 11 - 5 = 6$

↳ lower lim. of integration

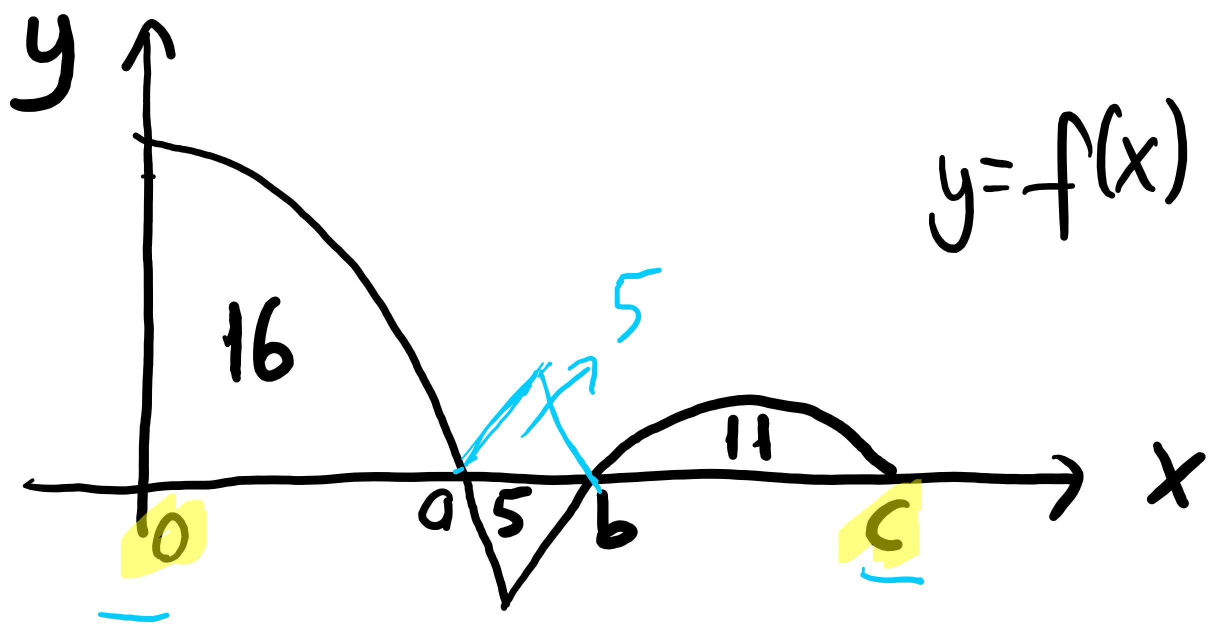
[check out the limits of integration]

b) $\int_a^c |f(x)|\,dx = 11 + 5 = 16$

a) 16
b) 6
c) −6

$|\ \ |$ → total area

$$\int_a^c f(x)\,dx \quad \text{vs.} \quad \int_a^c |f(x)|\,dx$$



Poll Q: $\displaystyle\int_0^c \left(2\cdot|f(x)| + 3\cdot f(x)\right) dx$

Net Area

$$\int_0^c 2\cdot|f(x)|\,dx \quad + \quad \int_0^c 3\cdot f(x)\,dx$$

$$2\cdot(16+5+11) \quad + \quad 3(\underbrace{16+11}_{27}-5)$$

$$2\cdot 32 \quad + \quad 3\cdot 22$$

$$64 \quad + \quad 66 \quad = 130$$

5.3. Fundamental Theorem of Calculus

$f(x)$

derivative

integral

$f'(x)$

FTC

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} R_n$$

slope of the tangent line

area under graph

FTC - Part 1

If f is continuous on $[a, b]$ and F is any antiderivative of f on $[a, b]$

Then
$$\int_a^b f(x)\,dx = F(b) - F(a) = F(x)\Big|_a^b$$

$\underbrace{f \to \text{function}}$ $\underbrace{F \to \text{antiderivative}}$

Recall:

$$\int_a^b f(x)\,dx$$ gives us a number

area under graph

$$\int f(x)\,dx = F(x) + \underline{\underline{C}}$$

gives us a family of functions

F is an antiderivative of f

FTC - Part 2

If f is continuous on $[a,b]$ then

$$\underbrace{A(x) = \int_a^x f(x)\,dt \quad \text{on } [a,b]}$$

The area function f. $(A(x))$

$$A'(x) = \frac{d}{dx}\left(\int_a^x f(x)\,dt\right) \Bigg\} \quad A'(x) = f(x)$$

$A(x)$ is an antiderivative of f on $[a,b]$

Exp 1) Evaluate the following integrals

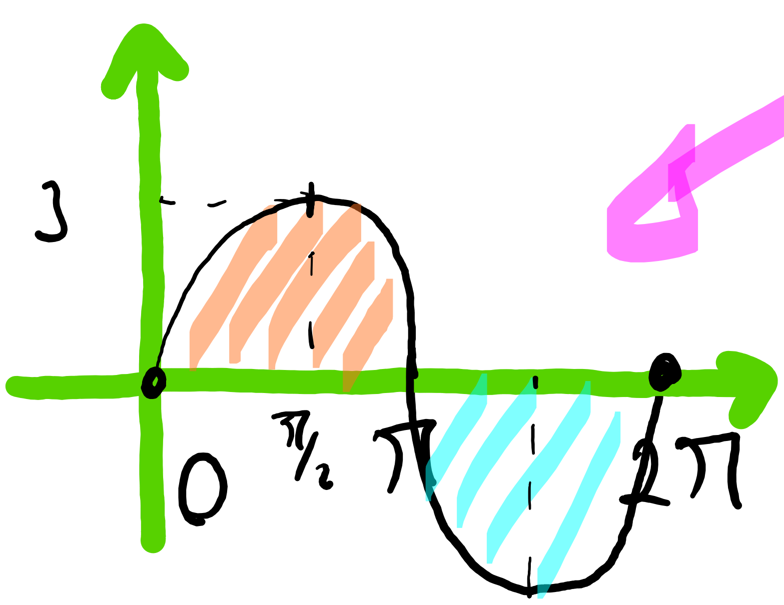
a) $\displaystyle\int_0^{2\pi} 3 \cdot \sin x\,dx = 0$

Recall:

$\sin 0 = 0$ $\quad \sin \pi/2 = 1$

$\sin \pi = 0$

$\sin 2\pi = 0$



$y = 3 \cdot \sin x$

$$\int_0^{2\pi} 3 \cdot \sin x\,dx = 3 \cdot \int_0^{2\pi} \sin x \cdot dx$$

$$= 3\left(-\cos x\right)\Big|_0^{2\pi} = 3(-\cos 2\pi + \cos 0)$$

$$= 3(-1+1) = 0$$