5.4 Group Activity Problems

4. Suppose f is an odd function, $\int_0^4 f(x)\,dx = 3$, and $\int_0^8 f(x)\,dx = 9$.

a. Evaluate $\int_{-4}^{8} f(x)\,dx$. **b.** Evaluate $\int_{-8}^{4} f(x)\,dx$.

5.4.4

a. Because $f(x)$ is odd, $\int_{-4}^{0} f(x)\,dx = -\int_0^4 f(x)\,dx$. We have

$$\int_{-4}^{8} f(x)\,dx = \int_{-4}^{0} f(x)\,dx + \int_0^8 f(x)\,dx = -\int_0^4 f(x)\,dx + \int_0^8 f(x)\,dx = -3+9=6.$$

b. $\int_{-8}^{4} f(x)\,dx = \int_{-8}^{0} f(x)\,dx + \int_0^4 f(x)\,dx = -9+3=-6.$

11–24. Symmetry in integrals *Use symmetry to evaluate the following integrals.*

16. $\int_{-\pi}^{\pi} t^2 \sin t\,dt$

20. $\int_{-1}^{1} (1 - |x|)\,dx$

22. $\int_{-\pi/4}^{\pi/4} \tan\theta\,d\theta$

5.4.16 Because $x^2\sin x$ is an odd function, $\int_{-\pi}^{\pi} x^2 \sin x\,dx = 0$

5.4.20 $\int_{-1}^{1}(1-|x|)\,dx = 2\int_0^1 (1-x)\,dx = 2\left(x-\frac{x^2}{2}\right)\Big|_0^1 = 2\left(1-\frac{1}{2}\right) = 1.$

5.4.22 Recall that the tangent function is an odd function, so the value of this integral is 0.

45. Explain why or why not Determine whether the following statements are true and give an explanation or counterexample.

a. If f is symmetric about the line $x = 2$, then $\int_0^4 f(x)\,dx = 2\int_0^2 f(x)\,dx$.

5.4.45

a. True. Because of the symmetry, the net area between 0 and 4 will be twice the net area between 0 and 2.