## 5.5 Group Activity Problems - Solutions


**Table 5.6 General Integration Formulas**

1. $\int \cos ax\,dx = \frac{1}{a}\sin ax + C$
2. $\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$
3. $\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + C$
4. $\int \csc^2 ax\,dx = -\frac{1}{a}\cot ax + C$
5. $\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax + C$
6. $\int \csc ax \cot ax = -\frac{1}{a}\csc ax + C$
7. $\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$
8. $\int b^x\,dx = \frac{1}{\ln b}b^x + C, b > 0, b \neq 1$
9. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$
10. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C, a > 0$
11. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C, a > 0$

**17–44. Indefinite integrals** *Use a change of variables or Table 5.6 to evaluate the following indefinite integrals. Check your work by differentiating.*

**17.** $\int 2x(x^2 - 1)^{99}\,dx$ **18.** $\int xe^{x^2}dx$

**5.5.17** Let $u = x^2 - 1$. Then $du = 2x\,dx$. Substituting yields $\int u^{99}\,du = \frac{u^{100}}{100} + C = \frac{(x^2-1)^{100}}{100} + C$.

**5.5.18** Let $u = x^2$. Then $du = 2x\,dx$, so $\frac{1}{2}du = x\,dx$. Substituting yields $\frac{1}{2}\int e^u\,du = \frac{1}{2}\cdot e^u + C = \frac{1}{2}\cdot e^{x^2} + C$.

**45–74. Definite integrals** *Use a change of variables or Table 5.6 to evaluate the following definite integrals.*

**64.** $\displaystyle\int_0^{\ln 4} \frac{e^x}{3 + 2e^x}\,dx$

5.5.64 Let $u = 3 + 2e^x$, so that $du = 2e^x\,dx$. Substituting yields $\displaystyle\frac{1}{2}\int_5^{11} \frac{1}{u}\,du = \frac{1}{2}\ln|u|\Big|_5^{11} = \frac{\ln(11/5)}{2}$.

**70.** $\displaystyle\int_{-1}^{1} (x - 1)(x^2 - 2x)^7\,dx$

5.5.70 Let $u = x^2 - 2x$. Then $du = 2(x-1)\,dx$. Also note that when $x = -1$ we have $u = 3$ and when $x = 1$ we have $u = -1$. Substituting yields $\displaystyle\frac{1}{2}\int_3^{-1} u^7\,du = \frac{1}{16}\left(u^8\right)\Big|_3^{-1} = \frac{1}{16}\left(1 - 3^8\right) = -\frac{6560}{16} = -410.$

**95. Explain why or why not** Determine whether the following statements are true and give an explanation or counterexample.

**c.** $\displaystyle\int \sin 2x\,dx = 2\int \sin x\,dx.$

**d.** $\displaystyle\int (x^2 + 1)^9 dx = \frac{(x^2 + 1)^{10}}{10} + C.$

c. False. If this were true, then $\sin 2x$ and $2\sin x$ would have to differ by a constant, which they do not. In fact, $\sin 2x = 2\sin x\cos x$.

d. False. The derivative of the right hand side is $(x^2+1)^9 \cdot 2x$ which is not the integrand on the left hand side.

**96–98. Areas of regions** *Find the area of the following regions.*

**96.** The region bounded by the graph of $f(x) = \dfrac{x}{\sqrt{x^2 - 9}}$ and the $x$-axis between $x = 4$ and $x = 5$

5.5.96 $\displaystyle A(x) = \int_4^5 \frac{x}{\sqrt{x^2-9}}\,dx$. Let $u = x^2 - 9$, so that $du = 2x\,dx$. Also, when $x = 4$ we have $u = 7$ and when $x = 5$ we have $u = 16$. Substituting yields $\displaystyle\frac{1}{2}\int_7^{16} u^{-1/2}\,du = \sqrt{u}\Big|_7^{16} = 4 - \sqrt{7}.$