Math 135: Calculus I, Exam #1, Book#1 of 2

Name: ______________________________

ID# (last 4 digits): ____________ Section: ____________

- Problems #1 – #8 are marked as "no partial credit". For these problems, you are not required to show work, and any scratch work will not be considered. You will be awarded none or all of the points, depending only on whether your answer is exactly correct.
- Problems #9 – #10 are marked as "partial credit" For these problems, you are required to show work, and you will be awarded points based on your work. Your work must be written clearly using proper notation. Answers must be justified using techniques that have been taught in this course, and answers without such justification may receive less than full credit – or no credit at all – even if the answer is correct.
- This exam is closed book. Calculators, electronic devices, notes, books, formula sheets, and other outside materials are not allowed. Phones must be turned off and put away.
- Unless otherwise stated, give exact answers: e.g., write π and $\sqrt{2}$ instead of 3.14 and 1.41. However, when an expression simplifies to a well-known value, you must use that value. For example, you must write 1 instead of e^0, and you must write $\frac{1}{2}$ instead of $\cos(\frac{\pi}{3})$.

Problem	Points	Score
1.	5	
2.	5	
3.	5	
4.	5	
5.	5	
6.	5	
7.	5	
8.	5	
9.	12	
10.	12	
11.	12	
12.	12	
13.	12	
Total:	100	

For problems #1 – #8, write your final answer in the appropriate box below.

Problem	Final Answer
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	

For problems #1 – #8, you are not required to show work, and any work you do write will not be graded. Write your final answer in the table on Page #2 of the exam.

1. Evaluate the limit or determine that it does not exist. $\lim_{x\to 0}\left(\frac{(2x-3)^2-9}{x}\right)$.

$$\lim_{x\to 0}\left(\frac{4x^2-12x+\cancel{9}-\cancel{9}}{x}\right)=\lim_{x\to 0}\left(\frac{4\cancel{x}(x-3)}{\cancel{x}}\right)$$

$$\lim_{x\to 0} 4(x-3) = 4(0-3) = -12$$

2. Evaluate the limit or determine that it does not exist. $\lim_{x\to 0}\left(\frac{\sin(12x)}{-7x}\right)$.

$$\lim_{x\to 0}\left(\frac{\sin(12x)}{-7x}\cdot\frac{12}{12}\right)=\lim_{x\to 0}\left(\frac{\sin 12x}{12x}\cdot\frac{12}{-7}\right)$$

$$=\lim_{x\to 0}\left(\frac{\sin(12x)}{12x}\right)^{\to 1}\cdot\lim_{x\to 0}\left(\frac{12}{-7}\right)=\frac{-12}{7}$$

Recall: $\lim_{x\to 0}\frac{\sin(cx)}{cx}=1$

3. Evaluate the limit or determine that it does not exist.

$$\lim_{x\to 16}\left(\frac{\sqrt{x}-4}{\frac{1}{x}-\frac{1}{16}}\right)$$

$$\lim_{x\to 16}\left(\frac{\sqrt{x}-4}{\frac{1}{x}\underset{(16)}{}-\frac{1}{16}\underset{(x)}{}}\right)=\lim_{x\to 16}\left(\frac{\sqrt{x}-4}{\frac{16-x}{16x}}\right)$$

$$\lim_{x\to 16}\left(\frac{\sqrt{x}-4}{\frac{16-x}{16x}}\cdot\frac{(\sqrt{x}+4)}{(\sqrt{x}+4)}\right)=\lim_{x\to 16}\frac{\cancel{x-16}^{\,-1}}{\frac{\cancel{(16-x)}\cdot(\sqrt{x}+4)}{16x}}$$

$$=\lim_{x\to 16}\frac{-16x}{\sqrt{x}+4}=\frac{-16\cdot 16}{\sqrt{16}+4}=\frac{-16\cdot 16}{8}=-32$$

4. Calculate $g'(x)$. After calculating the derivative, do not simplify your answer.

$$g(x)=3x^3-\frac{1}{3x}-4\sqrt{x}-5\pi^2$$

$$g(x)=3x^3-\frac{1}{3}\cdot x^{-1}-4\cdot x^{1/2}-5\pi^2$$

$$g'(x)=9x^2-\frac{1}{3}(-1)\cdot x^{-2}-4\cdot\frac{1}{2}\cdot x^{-1/2}$$

OR

$$g'(x)=9x^2+\frac{1}{3}\cdot x^{-2}-2x^{-1/2}$$

5. Calculate $h'(x)$. After calculating the derivative, do not simplify your answer.

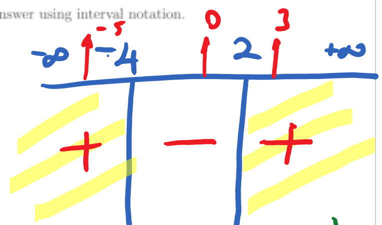
$$h(x) = \ln(3x - \tan(x))$$

$$h'(x) = \frac{(3x - \tan(x))'}{(3x - \tan(x))} = \boxed{\frac{3 - \sec^2 x}{3x - \tan(x)}}$$

6. Solve the inequality $\frac{5x - 10}{x + 4} > 0$. Write your answer using interval notation.

cut points: $x = 2, -4$

test points: $x = -5, 0, 3$



test the points:

$x = -5 \Rightarrow \frac{5(-5) - 10}{-5 + 4} = \frac{-35}{-1} > 0$

$(-\infty, -4) \cup (2, \infty)$

(full credit if student used and, or, "comma" instead $\cup$)

7. Calculate $g'(x)$. After calculating the derivative, do not simplify your answer.

$$g(x) = \sqrt{\sin(8+x^3)} = (\sin(8+x^3))^{1/2}$$

$$g'(x) = \frac{1}{2} \cdot \sin(8+x^3)^{-1/2} \cdot \cos(8+x^3) \cdot \underbrace{3x^2}_{(8+x^3)'}$$

8. Find the values of c and d that make f continuous for all x or determine that no such values of c and d exist.

$$f(x) = \begin{cases} \frac{\sin(6x)}{cx} & , \quad x < 0 \\ d & , \quad 0 \le x \le 6 \\ \frac{x^2 - 6x}{x-6} & , \quad x > 6 \end{cases}$$

check continuity at transition points: $x = 0, 6$; left-limit, right-limit and $f(x)$ values at these points must be equal.

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0) \qquad \lim_{x \to 0^-} \frac{\sin(6x)}{cx} = d$$

$$\lim_{x \to 6^-} f(x) = \lim_{x \to 6^+} f(x) = f(6) \qquad \lim_{x \to 6^+} \frac{x(x-6)}{x-6} = d$$

$$\lim_{x \to 0^-} \frac{\sin(6x)}{cx} = 6 \Rightarrow \frac{6}{c} = 6 \Rightarrow c = 1 \qquad \boxed{6 = d}$$

For problems #9 – #13, you must show all work, and your work will be graded. Your work should be clear and use proper notation.

9. You must use the limit definition of derivative and proper notation to receive full credit.

Let $f(x) = \sqrt{11x}$. Calculate $f'(x)$.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{11(x+h)}-\sqrt{11x}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{11(x+h)}-\sqrt{11x}}{h} \cdot \frac{\sqrt{11(x+h)}+\sqrt{11x}}{\sqrt{11(x+h)}+\sqrt{11x}}$$

$$= \lim_{h\to 0} \frac{(11(x+h)-11x)}{h(\sqrt{11(x+h)}+\sqrt{11x})}$$

$$= \lim_{h\to 0} \frac{11h}{h(\sqrt{11(x+h)}+\sqrt{11x})} = \lim_{h\to 0} \frac{11}{\sqrt{11(x+h)}+\sqrt{11x}}$$

$$= \frac{11}{\sqrt{11(x+0)}+\sqrt{11x}} = \frac{11}{2\sqrt{11x}}$$

10. Find the absolute minimum value and absolute maximum value of $f(x) = x^4 - 2x^2$ on $[-2, 0]$.

absolute minimum value:	-1
absolute maximum value:	8

Procedure for finding absolute extrema.

Step 1) $f'(x) = 4x^3 - 4x = 4x(x^2-1)$
$= 4x(x-1)(x+1)$

$f'(x) = 0 = 4x(x-1)(x+1) \Rightarrow x = 0, 1, -1$

$f'(x)$ DNE no value of x makes $f'(x)$ DNE.

Critical # of f on [-2,0] doesn't include $x=1$!

Step2)

x	$f(x) = x^4 - 2x^2$
0	0
-1	-1 → abs. min
-2	8 → abs. max

Step3) The abs. max value is 8. The abs. min value is -1.

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Problem	Points	Score
1.	5	
2.	5	
3.	5	
4.	5	
5.	5	
6.	5	
7.	5	
8.	5	
9.	12	
10.	12	
11.	12	
12.	12	
13.	12	
Total:	100	

For problems #11 – #13, you must show all work, and your work will be graded. Your work should be clear and use proper notation.

11. Find all points on the graph of $y = x \ln x$ where the tangent line is horizontal.

Use product rule

$$y' = (x)' \cdot \ln x + (x) \cdot (\ln x)'$$

$$= 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$$

The tangent line is horizontal when $y' = 0$

$$y' = \ln x + 1 = 0$$

$$\ln x = -1$$

$$x = e^{-1}$$

$$x = \frac{1}{e}$$

when $x = e^{-1}$

$$y = e^{-1} \cdot \ln(e^{-1})$$

$$y = e^{-1} \cdot (-1)$$

$$y = \frac{-1}{e}$$

Therefore; the point $(x, y) = \left(\frac{1}{e}, \frac{-1}{e}\right)$ is the point on the graph of $y = x \cdot \ln x$ at which the tangent line is horizontal.

12. Find the value of k that makes f continuous at $x = -2$ or determine that no such value of k exists. You must use limits and proper notation to receive full credit.

$$f(x) = \begin{cases} 3x + k & , \ x < -2 \\ 4 & , \ x = -2 \\ kx^3 + 3 & , \ x > -2 \end{cases}$$

value of k:	No such value of k exists

If f is to be continuous at $x = -2$, Then left-limit, right-limit, and $f(-2)$ must be equal.

$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} (3x+k) = 3(-2)+k = -6+k$

$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} (kx^3+3) = k(-2)^3+3 = -8k+3$

$f(-2) = 4$

However; $-6+k = 4$ yields $k = 10$
$-8k+3 = 4$ yields $k = -1/8$

Since, k can't be 10 and $-1/8$ at the same time, there's no value of k exists that makes f continuous at $x = -2$.

13. Find an equation of the line normal to the graph of $y = 2x^4 + x^2 - 3$ at $x = -1$. You may provide any form of the equation of a line.

Let $y=f(x)=2x^4+x^2-3$

The derivative at a general point is:

$$y'=f'(x)=8x^3+2x$$

The derivative at $x=-1$ is:

$$f'(-1)=8(-1)^3+2(-1)=-8-2=-10$$

Therefore, the slope of the tangent line at $x=-1$ is -10.

The slope of the normal line at $x=-1$ is $\frac{-1}{m_{tan}}=\frac{-1}{-10}=\frac{1}{10}$.

The normal line must pass through:

$(-1,f(-1))=(-1,0)$ $\{f(-1)=8(-1)^3+2(-1)\}$

The equation of the normal line is:

$$y-0=\frac{1}{10}(x-(-1))$$

OR $y=\frac{1}{10}(x+1)$

9) Let $f(x)=\sqrt{11x}$, calculate $f'(x)$ by using the limit definition of derivative.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

1pt for limit def. of derivative

$$=\lim_{h\to 0}\frac{\sqrt{11(x+h)}-\sqrt{11x}}{h}$$

2pts. for replacing $f(x)$, $f(x+h)$ correctly

$$=\lim_{h\to 0}\frac{\sqrt{11(x+h)}-\sqrt{11x}}{h}\cdot\frac{\sqrt{11(x+h)}+\sqrt{11x}}{\sqrt{11(x+h)}+\sqrt{11x}}$$

3 pts for multiplying by the conjugate

$$=\lim_{h\to 0}\frac{(11(x+h)-11x)}{h(\sqrt{11(x+h)}+\sqrt{11x})}$$

2pts. for rationalizing the numerator correctly.

$$=\lim_{h\to 0}\frac{\cancel{11x}+11h-\cancel{11x}}{h(\sqrt{11(x+h)}+\sqrt{11x})}$$

1 pt. for canceling out $11x$ and $-11x$.

$$=\lim_{h\to 0}\frac{11\cancel{h}}{\cancel{h}(\sqrt{11(x+h)}+\sqrt{11x})}$$

1pt. for canceling out h

$$=\lim_{h\to 0}\frac{11}{\sqrt{11(x+h)}+\sqrt{11x}}=\frac{11}{\sqrt{11(x+0)}+\sqrt{11x}}$$

1 pt. for substituting 0 for h.

$$=\frac{11}{2\sqrt{11x}}$$

1 pt. for correct final answer

Award 0 points if student does not clearly use the limit definition.

10) Find the abs. extrema of $f(x)=x^4-2x^4$ on $[-2,0]$.

Procedure for finding absolute extrema.

Step 1) $f'(x) = \underbrace{4x^3-4x}_{\text{1pt}} = \underbrace{4x(x^2-1)}_{\text{1pt.}} = \underbrace{4x(x-1)(x+1)}_{\text{1pt.}}$

find $f'(x)$

$f'(x)=0 \Rightarrow 4x(x-1)(x+1)=0 \Rightarrow \underbrace{x=0,1,-1}$

4pts (solutions to $f'(x)=0$)

Step 2) Evaluate $f(x)$ at endpoints and critical #s in $[-2,0]$, which means $\underline{x=-2,0;\ x=0,-1}$

2pts (identify correct critical #s in $[-2,0]$ and endpoints)

Critical # of f on $[-2,0]$ don't include $\cancel{x=1}$

calculate y-values:

$f(0)=0$

$f(-1)=-1$

$f(-2)=8$

2pts. for correct y-values

deduct 1 pt. for 1 arithmetic mistake w/ y-values

deduct 2pts for multiple arithmetic mistakes

Step 3) Compare $\underline{\text{values}}$ in step 2

The absolute max. value is 8.

The absolute min. value is −1.

1 pt. for final answer in "value" form.

(Don't double penalize for arithmetic mistakes)