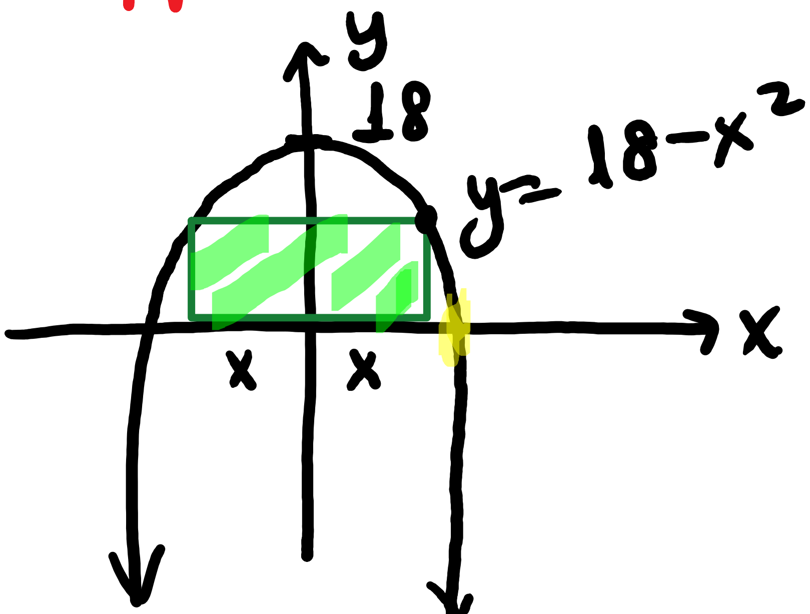
Q8) Find the length and width of the rectangle with the largest area whose lower two vertices lie on the x-axis and whose upper two vertices lie on the graph of $y=18-x^2$.



$A_{rectangle} = l \cdot w$

Let $2x$ be the length

y be the width

$(y=18-x^2)$ → constraint

$A(x,y) = 2x \cdot y$

$A(x) = 2x\cdot(18-x^2) = 36x-2x^3$ Obj. Function

$A'(x)=0$ or DNE

$A'(x) = 36-6x^2 = 0 \Rightarrow 36-6x^2=0 \Rightarrow 36=6x^2$

$\Rightarrow x = \mp\sqrt{6}$

(x can't be neg.)

$x=\sqrt{6}$

To justify $x=\sqrt{6}$ really produces the MAX area:

$A''(x) = -12x$ (concave down)

$x=\sqrt{6}$ is actually global max.

$l = 2x = \boxed{2\cdot\sqrt{6}}$, $w = 18-x^2 \Rightarrow w = 18-(\sqrt{6})^2$
$= 18-6 = \boxed{12}$

9) path of the particle is modeled by:

$$x^2+xy+2y^2=16$$

y-coordinate is decreasing at 5 in/sec. when the particle passes $(2,-3)$

At what rate is its x-coordinate changing?

Given: $x^2+xy+2y^2=16$

$\frac{dy}{dt}=-5\frac{in}{sec.}$ at $(2,-3)=(x,y)$

Asked: $\frac{dx}{dt}=?$

$$2x\cdot\frac{dx}{dt}+\frac{dx}{dt}\cdot y+x\cdot\frac{dy}{dt}+4y\cdot\frac{dy}{dt}=0$$

$$2\cdot 2\cdot\frac{dx}{dt}+\frac{dx}{dt}\cdot(-3)+2\cdot(-5)+4\cdot(-3)\cdot(-5)=0$$

$$\frac{dx}{dt}-10+60=0$$

$$\boxed{\frac{dx}{dt}=-50\frac{in}{sec.}}$$

Q10) $p(x) = 96 - 4x - x^2$

Use marginal analysis to estimate $R'(x)$ (MR) derived from producing the 5th unit.

$R(x) = p(x) \cdot x = 96x - 4x^2 - x^3$

$R'(x) = 96 - 8x - 3x^2$

$R'(4) = \$16$

10b) $\lim_{x \to \infty} \left(\frac{2x^2-5x+7}{5x^3-2x-10} \right) \to$ div. by x^3

$$\lim_{x \to \infty} \left(\frac{\frac{2x^2-5x+7}{x^3}}{\frac{5x^3-2x-10}{x^3}} \right) = \lim_{x \to \infty} \left(\frac{\cancel{\frac{2}{x}} - \cancel{\frac{5}{x^2}} + \cancel{\frac{7}{x^3}}}{5 - \cancel{\frac{2}{x^2}} - \cancel{\frac{10}{x^3}}} \right)$$

$$= \lim_{x \to \infty} \left(\frac{0}{5-0} \right) = 0$$

Q11) Find the eq. of the normal line to $3x^2-10xy+7y^2=3$ at $P(5,2)$

Implicitly differentiate the equation with respect to x to obtain:

$$6x-10x\cdot\frac{dy}{dx}-10\cdot y+14y\cdot\frac{dy}{dx}=0$$

substitute $P(5,2)$ into the equation:

$$6\cdot 5-10\cdot 5\cdot\frac{dy}{dx}-10\cdot 2+14\cdot 2\cdot\frac{dy}{dx}=0$$

$$30-50\cdot\frac{dy}{dx}-20+28\frac{dy}{dx}=0$$

$$10-22\cdot\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{10}{22}=\frac{5}{11}$$

$m_{tan}=\frac{5}{11}$, $m_{normal}=-\frac{11}{5}$

Equation of the line normal to curve at $(5,2)$:

$$y-2=-\frac{11}{5}(x-5)$$

OR $y=-\frac{11}{5}x+13$

OR $11x+5y-65=0$

Q12) $f(x)=\frac{x^3}{x^2-9}$, $f'(x)=\frac{x^2(x^2-27)}{(x^2-9)^2}$, $f''(x)=\frac{18x(x^2+27)}{(x^2-9)^3}$

Domain of f: $(-\infty,-3)$, $(-3,3)$, $(3,\infty)$

$x^2-9\neq 0 \Rightarrow x\neq \mp 3$

Vertical Asymptote: $f(x)$ is undefined at $x=\mp 3$,
Direct substitution of $x=\mp 3$ in $f(x)$ gives:
"$\frac{\text{non zero \#}}{0}$", which indicates that both one sided limits are infinite.

Horizontal Asymptote: NONE

$\lim_{x\to-\infty} f(x)=-\infty$, $\lim_{x\to\infty} f(x)=-\infty$

Increasing / Decreasing:
Find first-order critical numbers of $f(x)$ by $f'(x)=0$

$f'(x)=0=\frac{x^2(x^2-27)}{(x^2-9)^2} \Rightarrow x^2=0$ or $x^2-27=0$

$x=0$ $\qquad$ $x=\mp\sqrt{27}$

$x=\mp 3\sqrt{3}$

when constructing the sign charts, also include the x-coordinates of V.A:

sign chart for $f'(x)$

	$-\infty$		$-\sqrt{27}$		-3		0		3		$\sqrt{27}$		$+\infty$
sign of $f'(x)$		+		−		−		−				+	
increasing/decreasing		↗		↘		↘		↘		↘		↗	

local max at $x = -\sqrt{27}$

local min at $x = \sqrt{27}$

Note: $f'(x) = \dfrac{x^2(x^2-27)}{(x^2-9)^2}$

always positive, check the sign of (x^2-27) ONLY!

Increasing: $(-\infty, -3\sqrt{3}),\ (3\sqrt{3}, \infty)$

Decreasing: $(-3\sqrt{3}, -3),\ (-3, 0),\ (0, 3),\ (3, 3\sqrt{3})$

OR $(-3\sqrt{3}, 3\sqrt{3})$

Concave Up/Down:

Second-order critical numbers for $f(x)$:

$f''(x)=0$

$$f''(x)=\frac{18x(x^2+27)}{(x^2-9)^3}=0 \Rightarrow 18x=0 \text{ or } x^2+27=0$$

$x=0$ None

When constructing the sign charts, also include the x-coordinates of V.A:

sign chart for $f''(x)$

f''	$-\infty$ -4 -3	-2 0	2 3	4 $+\infty$
sign of $f''(x)$	$-$	$+$	$-$	$+$
concave up/down				

-4, -2, 2, 4 → test points

V.A: $x=-3$ Inflection Point   V.A: $x=3$

Concave Up: $(-3,0)$, $(3,\infty)$

Concave Down: $(-\infty,-3)$, $(0,3)$

X-coordinates of inflection point: $x=0$ ONLY since $x=\mp 3$ are not in the domain of $f(x)$.