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| **Student:** ____________________<br>**Date:** ____________________ | **Instructor:** Sheila Tabanli<br>**Course:** Math 136 | **Assignment:** Final Exam - Part1 |

1. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

**You must show work and submit your scrap work to earn credit for this question. Any discrepancy between the submitted work below and the scrap work will result in deduction of points.**

Use n = 4 to approximate the value of the integral by the following methods:
(a) the trapezoidal rule,
(b) Simpson's rule,
(c) Find the exact value by integration,
(d) Answer the conceptual question about Simpson's rule.

$$\int_0^2 \left(3x^2 + 2\right)\,dx$$

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(a) Use the trapezoidal rule to approximate the integral.

$$\int_0^2 \left(3x^2 + 2\right)\,dx \approx \underline{\qquad\qquad}$$

(Round the final answer to three decimal places as needed. Round all intermediate values to four decimal places as needed.)

(b) Use Simpson's rule to approximate the integral.

$$\int_0^2 \left(3x^2 + 2\right)\,dx \approx \underline{\qquad\qquad}$$

(Round the final answer to three decimal places as needed. Round all intermediate values to four decimal places as needed.)

(c) Find the exact value of the integral by integration.

$$\int_0^2 \left(3x^2 + 2\right)\,dx = \underline{\qquad\qquad}$$

(Type an integer or a decimal. Do not round.)

(d) Answer the conceptual question about Simpson's rule.

When using Simpson's rule to approximate a definite integral, what is true about the number of partitions?

- **A.** The number of partitions must be an odd number.
- **B.** The number of partitions must be a multiple of 4.
- **C.** The number of partitions must be an even number.
- **D.** The number of partitions must be a multiple of 3.

Show your work below.

2. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**
**You must show work and submit your scrap work to earn credit for this question. Any discrepancy between the submitted work below and the scrap work will result in deduction of points.**

Solve the initial value problem.

$y'(t) + 4y = 3,\ y(0) = 1$

- **A.** $y = \frac{3}{4}e^{4t} + \frac{1}{4}$
- **B.** $y = \frac{3}{4}e^{-4t} + \frac{1}{4}$
- **C.** $y = \frac{1}{4}e^{-4t} + \frac{3}{4}$
- **D.** $y = \frac{1}{4}e^{4t} + \frac{3}{4}$

Show your work below.

3. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

Verify that the given function is a solution of the differential equation that follows it.

$u(t) = C_1 t^4 + C_2 t^2$; $3t^2 u''(t) - 15tu'(t) + 24u(t) = 0$

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What is the best first step in verifying the solution?

- **A.** Solve for $u'(t)$ in the differential equation.
- **B.** Integrate $u''(t)$ in the differential equation.
- **C.** Differentiate the given function.
- **D.** Integrate the given function.

Perform the operation.

$u'(t) =$ ____________

What is the best next step?

- **A.** Solve for $u''(t)$ in the differential equation.
- **B.** Differentiate the first derivative.
- **C.** Substitute $u(t)$ and $u'(t)$ in the differential equation.
- **D.** Integrate both sides of the differential equation.

Perform the operation.

$u''(t) =$ ____________

What is the best next step?

- **A.** Substitute $u$, $u'$, and $u''$ into the differential equation.
- **B.** Integrate both sides of the differential equation.
- **C.** Differentiate both sides of the differential equation.
- **D.** Solve for $u(t)$ in the differential equation.

What is the result after simplifying?

- **A.** $u'(t) - 15u(t) + 24 = 0$
- **B.** $u(t) = C_1 t^4$
- **C.** $0 = 0$
- **D.** $C_2 t^2 = 0$

How has the solution been verified.

- **A.** It has been shown that substituting $u(t)$ and its first two derivatives into the differential equation results in a true statement.
- **B.** It has been shown that substituting $u(t)$ and its first two derivatives into the differential equation returns $u(t)$.
- **C.** It has been shown that substituting the first two derivatives of $u(t)$ into the differential equation returns $u(t)$.

**D.** It has been shown that integrating both sides of the differential equation twice returns u(t).

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4. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

**You must show work and submit your scrap work to earn credit for this question. Any discrepancy between the submitted work below and the scrap work will result in deduction of points.**

The following integral converges. Evaluate the integral.

$$\int_5^{\infty} \frac{7}{x^2+5x+6}\,dx$$

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$\int_5^{\infty} \frac{7}{x^2+5x+6}\,dx =$ ____________ (Type an exact answer.)

Show your work below.

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5. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

Use an appropriate substitution and then a trigonometric substitution to evaluate the integral.

$$\int \frac{dx}{x\sqrt{x^2-9}}$$

---

Which substitution transforms the given integral into one that can be evaluated directly in terms of $\theta$?

**A.** $x = 3\,\mathbf{sec}\,\theta$

**B.** $x = 3\,\mathbf{sin}\,\theta$

**C.** $x = 3\,\mathbf{tan}\,\theta$

Given the expression for x above, find dx in terms of $\theta$ and $d\theta$.

$dx =$ ____________ $d\theta$

$\int \frac{dx}{x\sqrt{x^2-9}} =$ ____________

6. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

Use the shell method to find the volume of the solid generated by revolving the region bound by $y = 2x$, $y = 0$, and $x = 3$ about the following lines.

**a.** The y-axis

**b.** The x-axis

**a.** Set up the integral that gives the volume of the solid generated by revolving around the y-axis.

$$\int_0^{\_\_\_\_\_\_} \_\_\_\_\_\_ \, dx$$

The volume of the given solid is ________ cubic units.
(Type an exact answer in terms of $\pi$.)

**b.** Set up the integral that gives the volume of the solid generated by revolving around the x-axis.

$$\int_0^{\_\_\_\_\_\_} \_\_\_\_\_\_ \, dy$$

The volume of the given solid is ________ cubic units.
(Type an exact answer in terms of $\pi$.)

7. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

Use the u-substitution method to evaluate the following integral.

$$\int_1^7 \frac{2 \ln x}{x} dx$$

Determine a change of variables from y to u. Choose the correct answer below.

- **A.** $u = \ln x$
- **B.** $u = \frac{2 \ln x}{x}$
- **C.** $u = 2 \ln x$
- **D.** $u = x$

Write the integral in terms of u.

$$\int_1^7 \frac{2 \ln x}{x} dx = \int_0^{\_\_\_\_\_\_} \_\_\_\_\_\_ \, du$$

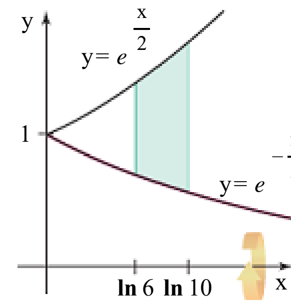
Evaluate the integral.

$$\int_1^7 \frac{2 \ln x}{x} dx = \_\_\_\_\_\_$$ (Type an exact answer.)

8. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

Let R be the region bounded by the following curves. Use the washer method to find the volume of the solid generated when R is revolved about the x-axis.

$y = e^{\frac{x}{2}}$, $y = e^{-\frac{x}{2}}$, $x = \ln 6$, $x = \ln 10$




Set up the integral that gives the volume of the solid. Use increasing limits of integration. Select the correct choice below and fill in the answer boxes to complete your choice.

(Type exact answers.)

○ **A.** $\int_{\_\_\_}^{\_\_\_} (\_\_\_\_\_\_)\,dy$

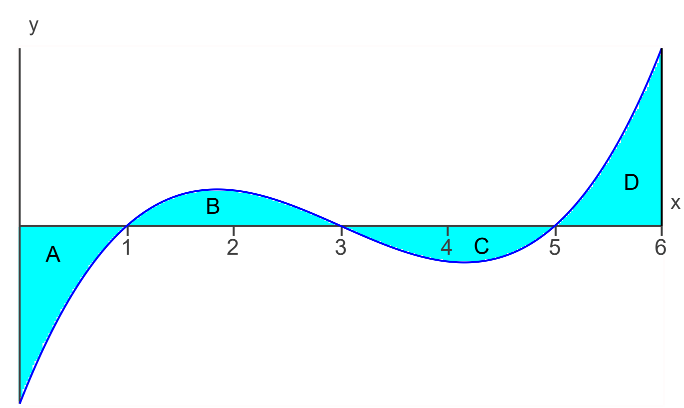
○ **B.** $\int_{\_\_\_}^{\_\_\_} (\_\_\_\_\_\_)\,dx$

The volume of the solid is ________ cubic units. (Type an exact answer.)

9. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

In the figure, the regions A, B, C and D bounded by the x-axis and the graph of f have the following properties:

- Region A and Region D each have area 250.
- Region B and Region C each have area 160.




Use the figure to calculate the following definite integral.

$$\int_6^1 (f(x) + 80)\,dx = \_\_\_\_\_\_\_\_$$

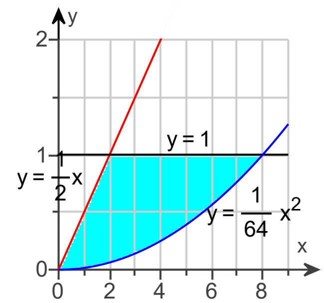
10. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

Find the area of the shaded region enclosed by the following functions.

$y = \frac{1}{2}x$

$y = 1$

$y = \frac{1}{64}x^2$




Set up the integral(s) that will give the area of the shaded region. Select the correct choice below and fill in any answer boxes to complete your choice.

○ **A.** $\int_0^{\_\_\_} [\_\_\_\_\_]\,dx + \int_{\_\_\_}^{1} [\_\_\_\_\_]\,dx$

○ **B.** $\int_0^{\_\_\_} [\_\_\_\_\_]\,dy + \int_{\_\_\_}^{8} [\_\_\_\_\_]\,dy$

○ **C.** $\int_0^{\_\_\_} [\_\_\_\_\_]\,dx$

○ **D.** $\int_0^{\_\_\_} [\_\_\_\_\_]\,dy$

Find the area by evaluating the integral.

________ (Type an integer or a simplified fraction.)

11. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

The integral can be found in more than one way. First use integration by parts, then integrate the result.

$$\int (x+3)(x+5)^2 dx$$

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Identify the best u and dv to use for integrating by parts. Choose the correct answer below.

- **A.** $u = 1,\ dv = (x+3)(x+5)^2 dx$
- **B.** $u = (x+5)^2,\ dv = (x+3)dx$
- **C.** $u = (x+3)(x+5)^2,\ dv = dx$
- **D.** $u = x+3,\ dv = (x+5)^2 dx$

Set up the integral using the integration-by-parts formula and u and dv found in the previous step.

$\_\_\_\_\_\_\_\_ - \int (\_\_\_\_\_\_\_\_)\, dx$

Evaluate the integral.

$\int (x+3)(x+5)^2 dx =$ ________

1. 12.25

   12

   12

   C. The number of partitions must be an even number.

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2. C. $y = \frac{1}{4} e^{-4t} + \frac{3}{4}$

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3. C. Differentiate the given function.

   $4C_1 t^3 + 2C_2 t$

   B. Differentiate the first derivative.

   $12C_1 t^2 + 2C_2$

   A. Substitute u, u′, and u′′ into the differential equation.

   C. $0 = 0$

   A.
   It has been shown that substituting u(t) and its first two derivatives into the differential equation results in a true statement.

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4. $7 \ln \frac{8}{7}$

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5. A. $x = 3 \sec \theta$

   $3 \sec \theta \tan \theta$

   $\frac{1}{3} \sec^{-1} \left( \frac{x}{3} \right) + C$

---

6. 3

   $2\pi x \cdot 2x$

   $36\pi$

   6

   $2\pi y \left( 3 - \frac{y}{2} \right)$

   $36\pi$

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7. A. $u = \ln x$

   $\ln 7$

   $2u$

   $(\ln 7)^2$

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8. B. $\displaystyle\int_{\ln 6}^{\ln 10} \left(\pi\left(e^{x} - e^{-x}\right)\right) dx$

$\frac{59\pi}{15}$

---

9. $-650$

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10. D. $\displaystyle\int_{0}^{1} \left[8\sqrt{y} - 2y\right] dy$

$\frac{13}{3}$

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11. D. $u = x + 3$, $dv = (x+5)^2 dx$

$\frac{1}{3}(x+3)(x+5)^3$

$\frac{1}{3}(x+5)^3$

$\frac{1}{3}(x+3)(x+5)^3 - \frac{1}{12}(x+5)^4 + C$

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