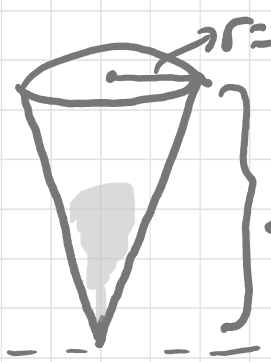
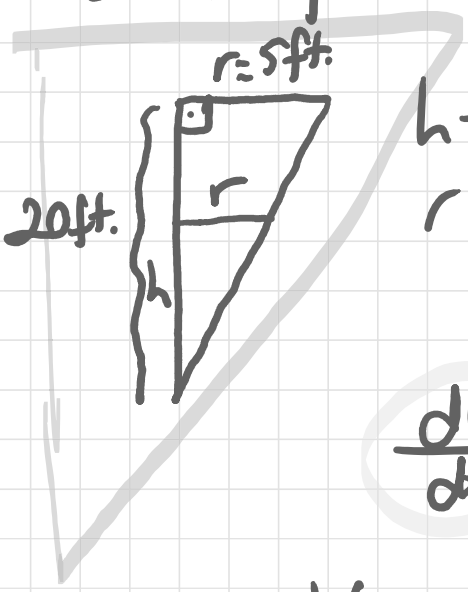
Exp4) The water level in a cone-shaped tank

A tank filled with water in the shape of an inverted cone 20 ft. high with a circular base (on top) whose radius is 5 ft. Water is running out of the bottom of the tank at the constant rate of 2 ft^3/min. How fast is the water level falling when the water is 8 ft. deep?



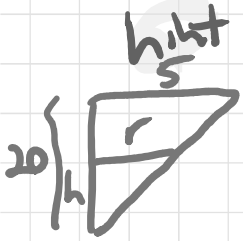


h → water level

r → radius of the cone that has water.

$\frac{dh}{dt} = ?$ when $h = 8ft$

$$\frac{dV}{dt} = 2\frac{ft^3}{min.}$$



base : height

$$\frac{5}{20} = \frac{r}{h}$$

$$5h = 20r$$

$$\frac{h}{4} = r$$

$$V = \frac{1}{3} \cdot \pi r^2 h$$

$$V = \frac{1}{3} \cdot \pi \cdot \left(\frac{h}{4}\right)^2 \cdot h$$

$$V = \frac{\pi}{48} \cdot h^3$$

$$V = \frac{\pi}{48} \cdot h^3$$ (diff. w/ respect to time)

given: $h = 8$ ft.

$$\frac{dV}{dt} = -2 \frac{ft^3}{min}$$

$$\frac{dV}{dt} = \frac{\pi}{48} \cdot 3h^2 \cdot \frac{dh}{dt}$$

$$-2 = \frac{\pi}{48} \cdot 3 \cdot 8 \cdot 8 \cdot \frac{dh}{dt}$$

$$\frac{-2}{4\pi} = \frac{4\pi}{4\pi} \cdot \frac{dh}{dt}$$

$$\frac{-2}{4\pi} = \frac{-1}{2\pi} \frac{ft}{min} = \frac{dh}{dt}$$

The water level is decreasing at the rate of $\frac{1}{2\pi} \frac{ft}{min}$ { recall $\pi \approx 3.14$; $\frac{1}{2\pi} \approx \frac{1}{6} \frac{ft}{min}$ }

(1 ft. per 6 minutes)