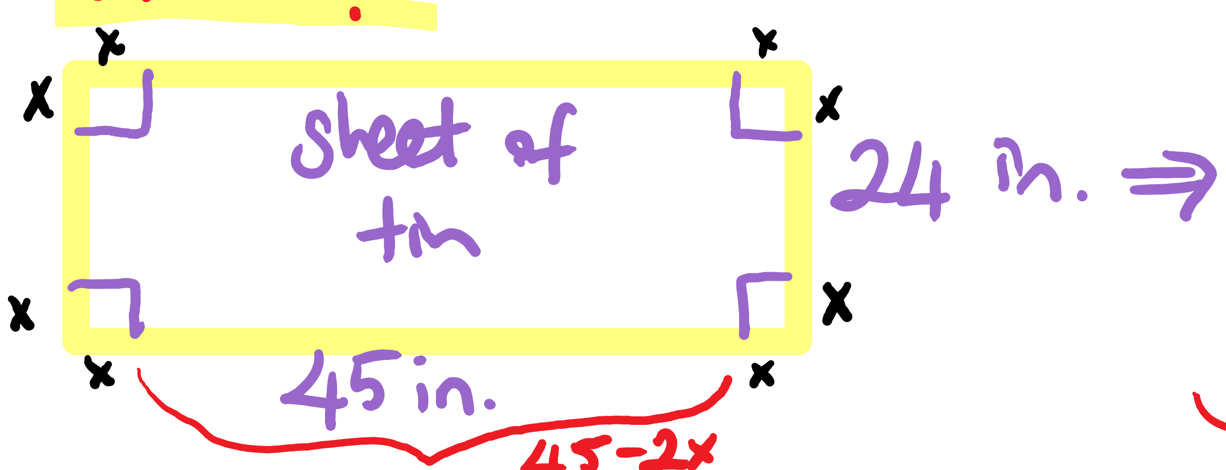
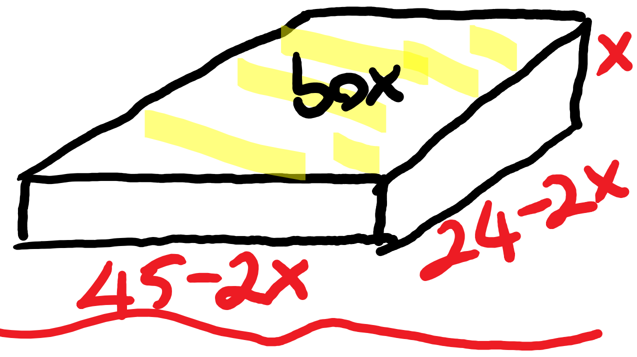
Exp. Max. a Volume

A carpenter wants to make an open-topped box of a rectangular sheet of tin 24 in. wide and 45 in. long. The carpenter plans to cut congruent squares out of each corner of the sheet and then bend and solder the edges of the sheet upward to form the sides of the box. For what dimensions does the box have the greatest possible volume?





let x be the congruent small squares that will be cut from each corner.

The goal is to find dims of the open box to produce the MAX VOLUME.

$$V(l,w,h) = l \cdot w \cdot h \quad \text{(Obj. F.)}$$

length → $45-2x$
w → $24-2x$
h → x

$$\underbrace{V(x) = (45-2x)\cdot(24-2x)\cdot x}$$

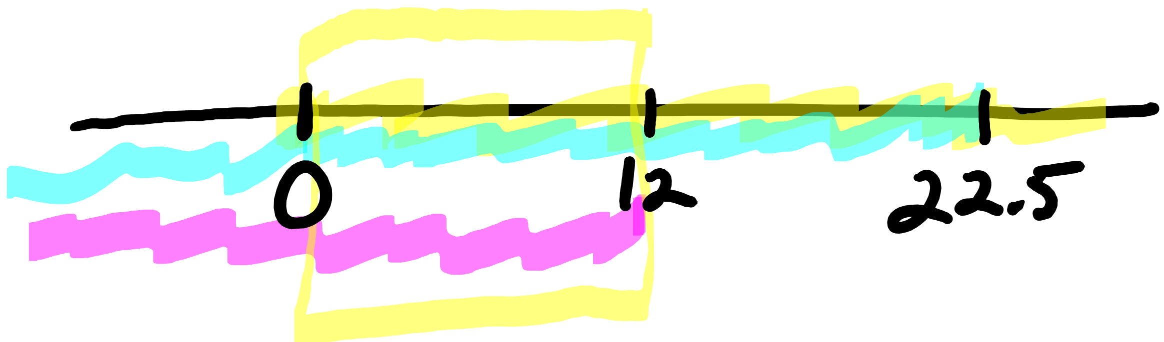
Constraint: Dims of the box should be greater or equal to 0.

$$x \geq 0 \Rightarrow x \geq 0 \qquad 1$$

$$45-2x \geq 0 \Rightarrow \frac{45}{2} \geq \frac{2x}{2} \Rightarrow 22.5 \geq x \qquad 2$$

$$24-2x \geq 0 \Rightarrow \frac{24}{2} \geq \frac{2x}{2} \Rightarrow 12 \geq x \qquad 3$$

$\underbrace{\text{Domain of } x}$



$$0 \leq x \leq 12$$

$$[0, 12]$$

$$V(x) = (45-2x)(24-2x)\cdot x$$
$$= 4x^3 - 138x^2 + 1080x$$

$$V'(x) = 12x^2 - 276x + 1080$$
$$= 12(x-18)(x-5)$$

domain of x
$[0,12]$

$V'(x) = 0$ or DNE

none

$\cancel{x=18}$ not in the domain

$x = 5$

Sign chart for $V'(x) = 12(x-18)(x-5)$

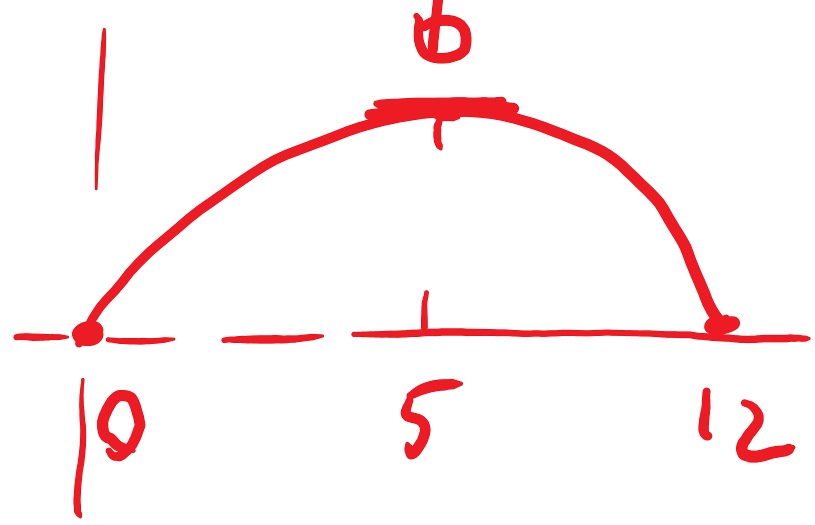
$V'(4) = +$

$V'(6) = -$

	0	4	5	6	12
sign of $V'(x)$		+		−	
incr/decr.		↗		↘	

local max at $x=5$

$$V(x) = (45-2x)\underline{(24-2x)}\cdot\underline{x}$$
$V(0) = 0$
$V(12) = 0$



$V(5) = + \quad + \quad + = +$

when $x=5$

$l = 45-2x$
$= 45-2\cdot 5$
$= 35$ in.

$w = 24-2x$
$= 24-2\cdot 5$
$= 14$ in.

$h = x$
$= 5$ in.

Dimensions of the open box with the greatest volume is $(l, w, h) = (35\text{ in}, 14\text{ in}, 5\text{ in})$