

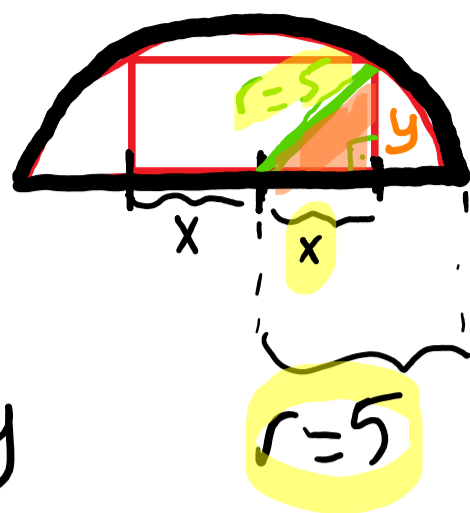
Rectangle Area Optimization

Tuesday, November 24, 2020 12:43 PM

Draw:

17. **Rectangles beneath a semicircle** A rectangle is constructed with its base on the diameter of a semicircle with radius 5 and its two other vertices on the semicircle. What are the dimensions of the rectangle with maximum area?

$r=5$



Obj. : MAX $A_{\text{rectangle}} = 2x \cdot y$

Constraints: right Δ : $x^2 + y^2 = 5^2$

constraint eq: $x^2 + y^2 = 5^2 \Rightarrow y^2 = 25 - x^2$

Obj. F: $A(x, y) = 2x \cdot y$ MAX
 $\hookrightarrow A(x), A(y)$

$y^2 = 25 - x^2 \Rightarrow y = \sqrt{25 - x^2}$ $\rightarrow (25 - x^2)^{1/2}$

Re-write obj. F: $A(x) = 2x \cdot \sqrt{25 - x^2}$ MAX

Critical P. of $A(x) \rightarrow$ MAX $A(x)$

$A'(x) = 0$ or DNE

$$A'(x) = 2 \cdot \sqrt{25 - x^2} + \cancel{2x} \cdot \frac{1}{\cancel{2}} (25 - x^2)^{-1/2} \cdot (-2x)$$

chain rule!

$$= \frac{2 \sqrt{25 - x^2}}{(\sqrt{25 - x^2})} - \frac{2x^2}{\sqrt{25 - x^2}} = \frac{2(25 - x^2) - 2x^2}{\sqrt{25 - x^2}}$$

