2.3 Group Activity Problems - Solutions

17. Suppose p and q are polynomials. If $\lim_{x \to 0} \frac{p(x)}{q(x)} = 10$ and $q(0) = 2$, find $p(0)$.

2.3.17 If p and q are polynomials then $\lim_{x \to 0} \frac{p(x)}{q(x)} = \frac{\lim_{x \to 0} p(x)}{\lim_{x \to 0} q(x)} = \frac{p(0)}{q(0)}$. Because this quantity is given to be equal to 10, we have $\frac{p(0)}{2} = 10$, so $p(0) = 20$.

32. $\lim_{h \to 0} \frac{100}{(10h - 1)^{11} + 2}$

2.3.32 $\lim_{h \to 0} \frac{100}{(10h - 1)^{11} + 2} = \frac{100}{(-1)^{11} + 2} = \frac{100}{1} = 100.$

37. $\lim_{x \to b} \frac{(x - b)^{50} - x + b}{x - b}$

2.3.37 $\lim_{x \to b} \frac{(x - b)^{50} - x + b}{x - b} = \lim_{x \to b} \frac{(x - b)^{50} - (x - b)}{x - b} = \lim_{x \to b} \frac{(x - b)((x - b)^{49} - 1)}{x - b} =$
$\lim_{x \to b} [(x - b)^{49} - 1] = -1.$

44. $\lim_{t \to 3} \left(\left(4t - \frac{2}{t-3}\right)(6 + t - t^2) \right)$

2.3.44 Expanding gives

$$\lim_{t\to 3}\left(\left(4t - \frac{2}{t-3}\right)(6+t-t^2)\right) = \lim_{t\to 3}\left(4t(6+t-t^2) - \frac{2(6+t-t^2)}{t-3}\right)$$
$$= \lim_{t\to 3}\left(4t(6+t-t^2) - \frac{2(3-t)(2+t)}{t-3}\right).$$

Now because $t - 3 = -(3-t)$, we have

$$\lim_{t\to 3}\left(4t(6+t-t^2) + 2(2+t)\right) = 12(6+3-9) + 2(2+3) = 10.$$

59. $\lim_{x \to 0} x \cos x$

2.3.59 $\lim_{x\to 0} x\cos x = 0 \cdot 1 = 0.$

64. $\lim_{w \to 3^-} \frac{|w-3|}{w^2 - 7w + 12}$

2.3.64 $\lim_{w\to 3^-} \frac{|w-3|}{w^2-7w+12} = \lim_{w\to 3^-} \frac{3-w}{(w-3)(w-4)} = -\lim_{w\to 3^-} \frac{1}{w-4} = 1.$

74. One-sided limits Let

$$f(x) = \begin{cases} 0 & \text{if } x \le -5 \\ \sqrt{25 - x^2} & \text{if } -5 < x < 5 \\ 3x & \text{if } x \ge 5. \end{cases}$$

Compute the following limits or state that they do not exist.

a. $\lim_{x \to -5^-} f(x)$ **b.** $\lim_{x \to -5^+} f(x)$ **c.** $\lim_{x \to -5} f(x)$

d. $\lim_{x \to 5^-} f(x)$ **e.** $\lim_{x \to 5^+} f(x)$ **f.** $\lim_{x \to 5} f(x)$

2.3.74

a. $\lim_{x \to -5^-} f(x) = \lim_{x \to -5^-} 0 = 0.$

b. $\lim_{x \to -5^+} f(x) = \lim_{x \to -5^+} \sqrt{25 - x^2} = \sqrt{25 - 25} = 0.$

c. $\lim_{x \to -5} f(x) = 0.$

d. $\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} \sqrt{25 - x^2} = \sqrt{25 - 25} = 0.$

e. $\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} 3x = 15.$

f. $\lim_{x \to 5} f(x)$ does not exist.

89. Finding a constant Suppose

$$g(x) = \begin{cases} x^2 - 5x & \text{if } x \le -1 \\ ax^3 - 7 & \text{if } x > -1. \end{cases}$$

Determine a value of the constant a for which $\lim_{x \to -1} g(x)$ exists and state the value of the limit, if possible.

2.3.89 In order for $\lim_{x \to -1} g(x)$ to exist, we need the two one-sided limits to exist and be equal. We have $\lim_{x \to -1^-} g(x) = \lim_{x \to -1^-} (x^2 - 5x) = 6$, and $\lim_{x \to -1^+} g(x) = \lim_{x \to -1^+} (ax^3 - 7) = -a - 7$. So we need $-a - 7 = 6$, so we require that $a = -13$. Then $\lim_{x \to -1} f(x) = 6$.

3.5 Group Activity Problems - Solutions

Practice Exercises

11–22. Trigonometric limits *Use Theorem 3.10 to evaluate the following limits.*

14. $\lim_{x \to 0} \frac{\sin 3x}{\tan 4x}$

16. $\lim_{\theta \to 0} \frac{\cos^2 \theta - 1}{\theta}$

18. $\lim_{\theta \to 0} \frac{\sec \theta - 1}{\theta}$

20. $\lim_{x \to -3} \frac{\sin(x + 3)}{x^2 + 8x + 15}$

21. $\lim_{x \to 0} \frac{\sin ax}{\sin bx}$, where a and b are constants with $b \neq 0$

22. $\lim_{x \to 0} \frac{\sin ax}{bx}$, where a and b are constants with $b \neq 0$

3.5.14 $$\lim_{x \to 0} \frac{\sin 3x}{\tan 4x} = \lim_{x \to 0} \frac{\sin 3x \cos 4x}{\sin 4x} = \lim_{x \to 0} \frac{\frac{3 \sin 3x}{3x} \cdot \cos 4x}{\frac{4 \sin 4x}{4x}} = \frac{3 \cdot 1 \cdot 1}{4 \cdot 1} = \frac{3}{4}.$$

3.5.16 $$\lim_{\theta \to 0} \frac{\cos^2 \theta - 1}{\theta} = \left(\lim_{\theta \to 0} (\cos \theta + 1) \right) \cdot \left(\lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta} \right) = 2 \cdot 0 = 0.$$

3.5.18 $$\lim_{\theta \to 0} \frac{\sec \theta - 1}{\theta} = \lim_{\theta \to 0} \frac{\frac{1}{\cos \theta} - 1}{\theta} = \lim_{\theta \to 0} \frac{1 - \cos \theta}{\theta \cos \theta} = \lim_{\theta \to 0} \frac{1}{\cos \theta} \cdot \lim_{\theta \to 0} \frac{1 - \cos \theta}{\theta} = 1 \cdot 0 = 0.$$

3.5.20 $\lim_{x \to -3} \frac{\sin(x+3)}{x^2+8x+15} = \lim_{x \to -3} \frac{\sin(x+3)}{(x+5)(x+3)} = \lim_{x \to -3} \frac{1}{x+5} \cdot \lim_{x \to -3} \frac{\sin(x+3)}{(x+3)} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$

3.5.21 $\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \lim_{x \to 0} \frac{a \sin ax}{ax} \cdot \frac{bx}{b \sin bx} = \frac{a}{b} \lim_{x \to 0} \frac{\sin ax}{ax} \cdot \lim_{x \to 0} \frac{bx}{\sin bx} = \frac{a}{b} \cdot 1 \cdot 1 = \frac{a}{b}.$

3.5.22 $\lim_{x \to 0} \frac{\sin ax}{bx} = \frac{a}{b} \lim_{x \to 0} \frac{\sin ax}{ax} = \frac{a}{b} \cdot 1 = \frac{a}{b}.$