2.5 Group Activity Problems

29. $\lim_{w \to \infty} \frac{15w^2 + 3w + 1}{\sqrt{9w^4 + w^3}}$

2.5.29 Note that for $w > 0$, $w^2 = \sqrt{w^4}$. We have

$$\lim_{w \to \infty} \frac{(15w^2 + 3w + 1)}{\sqrt{9w^4 + w^3}} \cdot \frac{1/w^2}{1/\sqrt{w^4}} = \lim_{w \to \infty} \frac{15 + (3/w) + (1/w^2)}{\sqrt{9 + (1/w)}} = \frac{15}{\sqrt{9}} = 5.$$

30. $\lim_{x \to -\infty} \frac{40x^4 + x^2 + 5x}{\sqrt{64x^8 + x^6}}$

2.5.30 Note that $\sqrt{x^8} = x^4$ (even for $x < 0$). We have

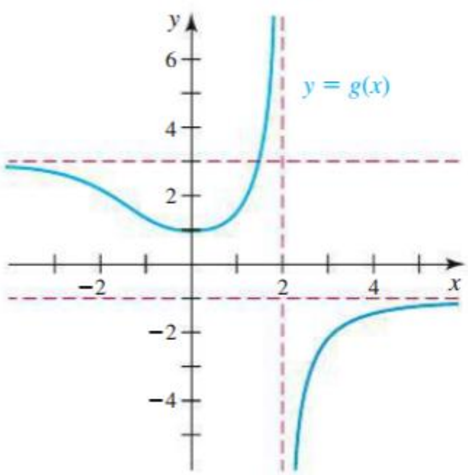
$$\lim_{x \to -\infty} \frac{(40x^4 + x^2 + 5x)}{\sqrt{64x^8 + x^6}} \cdot \frac{1/x^4}{1/\sqrt{x^8}} = \lim_{x \to -\infty} \frac{40 + (1/x^2) + (5/x^3)}{\sqrt{64 + (1/x^2)}} = \frac{40}{\sqrt{64}} = \frac{40}{8} = 5.$$

15. Suppose the function g satisfies the inequality $3 - \frac{1}{x^2} \leq g(x) \leq 3 + \frac{1}{x^2}$, for all nonzero values of x. Evaluate $\lim_{x \to \infty} g(x)$ and $\lim_{x \to -\infty} g(x)$.

2.5.15 Because $\lim_{x \to \infty} 3 - \frac{1}{x^2} = 3$ and $\lim_{x \to \infty} 3 + \frac{1}{x^2} = 3$, by the Squeeze Theorem we must have $\lim_{x \to \infty} g(x) = 3$.

Similarly, because $\lim_{x \to -\infty} 3 - \frac{1}{x^2} = 3$ and $\lim_{x \to -\infty} 3 + \frac{1}{x^2} = 3$, by the Squeeze Theorem we must have $\lim_{x \to -\infty} g(x) = 3$.

16. The graph of g has a vertical asymptote at $x = 2$ and horizontal asymptotes at $y = -1$ and $y = 3$ (see figure). Determine the following limits: $\lim_{x \to -\infty} g(x)$, $\lim_{x \to \infty} g(x)$, $\lim_{x \to 2^-} g(x)$, and $\lim_{x \to 2^+} g(x)$.



2.5.16 $\lim_{x \to -\infty} g(x) = 3$, $\lim_{x \to \infty} g(x) = -1$, $\lim_{x \to -2^-} g(x) = \infty$, $\lim_{x \to 2^+} g(x) = -\infty$.

Determine the following limits.

24. $\lim_{x \to -\infty} (2x^{-8} + 4x^3)$

26. $\lim_{x \to \infty} \frac{9x^3 + x^2 - 5}{3x^4 + 4x^2}$

28. $\lim_{x \to \infty} \frac{x^4 + 7}{x^5 + x^2 - x}$

32. $\lim_{x \to \infty} \frac{6x^2}{4x^2 + \sqrt{16x^4 + x^2}}$

2.5.24 $\lim_{x \to -\infty} (2x^{-8} + 4x^3) = 0 + \lim_{x \to -\infty} 4x^3 = -\infty.$

2.5.26 $\lim_{x \to \infty} \frac{(9x^3 + x^2 - 5)}{(3x^4 + 4x^2)} \cdot \frac{1/x^4}{1/x^4} = \lim_{x \to \infty} \frac{(9/x) + (1/x^2) - (5/x^4)}{3 + (4/x^2)} = \frac{0}{3} = 0.$

2.5.28 $\lim_{x \to \infty} \frac{(x^4 + 7)}{(x^5 + x^2 - x)} \cdot \frac{1/x^5}{1/x^5} = \lim_{x \to \infty} \frac{(1/x) + (7/x^5)}{1 + (1/x^3) - (1/x^4)} = \frac{0 + 0}{1 + 0 - 0} = 0.$

2.5.32 Note that $x^2 = \sqrt{x^4}$ for all x. We have

$$\lim_{x \to \infty} \frac{6x^2}{(4x^2 + \sqrt{16x^4 + x^2})} \cdot \frac{1/x^2}{1/x^2} = \lim_{x \to \infty} \frac{6}{(4 + \sqrt{16 + (1/x^2)})} = \frac{6}{4 + \sqrt{16}} = \frac{3}{4}.$$

37–50. Horizontal asymptotes *Determine* $\lim_{x\to\infty} f(x)$ *and* $\lim_{x\to-\infty} f(x)$ *for the following functions. Then give the horizontal asymptotes of f (if any).*

44. $f(x) = \dfrac{6x^2 + 1}{\sqrt{4x^4 + 3x + 1}}$

46. $f(x) = \dfrac{\sqrt{x^2 + 1}}{2x + 1}$

47. $f(x) = \dfrac{4x^3 + 1}{2x^3 + \sqrt{16x^6 + 1}}$

2.5.44 Note that for all x, $\sqrt{x^4} = x^2$. Then

$$\lim_{x\to\pm\infty} \frac{(6x^2+1)}{\sqrt{4x^4+3x+1}} \cdot \frac{1/x^2}{\sqrt{1/x^4}} = \lim_{x\to\pm\infty} \frac{6+(1/x^2)}{\sqrt{4+(3/x^3)+(1/x^4)}} = \frac{6}{\sqrt{4}} = 3.$$

So $y = 3$ is the only horizontal asymptote.

2.5.46 First note that $\sqrt{x^2} = x$ for $x > 0$, while $\sqrt{x^2} = -x$ for $x < 0$. Then $\lim_{x\to\infty} f(x)$ can be written as

$$\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{2x+1} \cdot \frac{1/\sqrt{x^2}}{1/x} = \lim_{x\to\infty} \frac{\sqrt{1+1/x^2}}{2+1/x} = \frac{1}{2}.$$

However, $\lim_{x\to-\infty} f(x)$ can be written as

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{2x+1} \cdot \frac{1/\sqrt{x^2}}{-1/x} = \lim_{x\to-\infty} \frac{\sqrt{1+1/x^2}}{-2-1/x} = -\frac{1}{2}.$$

2.5.47 First note that $\sqrt{x^6} = x^3$ if $x > 0$, but $\sqrt{x^6} = -x^3$ if $x < 0$. We have $\lim_{x\to\infty} \dfrac{4x^3+1}{(2x^3+\sqrt{16x^6+1})} \cdot \dfrac{1/x^3}{1/x^3} =$ $\lim_{x\to\infty} \dfrac{4+1/x^3}{2+\sqrt{16+1/x^6}} = \dfrac{4+0}{2+\sqrt{16+0}} = \dfrac{2}{3}$.

However, $\lim_{x\to-\infty} \dfrac{4x^3+1}{(2x^3+\sqrt{16x^6+1})} \cdot \dfrac{1/x^3}{1/x^3} = \lim_{x\to-\infty} \dfrac{4+1/x^3}{2-\sqrt{16+1/x^6}} = \dfrac{4+0}{2-\sqrt{16+0}} = \dfrac{4}{-2} = -2.$

So $y = \frac{2}{3}$ is a horizontal asymptote (as $x \to \infty$) and $y = -2$ is a horizontal asymptote (as $x \to -\infty$).

94. End behavior of exponentials Use the following instructions to determine the end behavior of $f(x) = \dfrac{4e^x + 2e^{2x}}{8e^x + e^{2x}}$.

a. Evaluate $\lim_{x \to \infty} f(x)$ by first dividing the numerator and denominator by e^{2x}.

b. Evaluate $\lim_{x \to -\infty} f(x)$ by first dividing the numerator and denominator by e^x.

c. Give the horizontal asymptote(s).

2.5.94

a. $$\lim_{x\to\infty} \frac{4e^x + 2e^{2x}}{8e^x + e^{2x}} = \lim_{x\to\infty} \frac{(4e^x + 2e^{2x})}{(8e^x + e^{2x})} \cdot \frac{1/e^{2x}}{1/e^{2x}} = \lim_{x\to\infty} \frac{(4/e^x) + 2}{(8/e^x) + 1} = 2.$$

b. $$\lim_{x\to-\infty} \frac{4e^x + 2e^{2x}}{8e^x + e^{2x}} = \lim_{x\to-\infty} \frac{(4e^x + 2e^{2x})}{(8e^x + e^{2x})} \cdot \frac{1/e^x}{1/e^x} = \lim_{x\to-\infty} \frac{4 + 2e^x}{8 + e^x} = \frac{1}{2}.$$

c. The lines $y = 2$ and $y = \frac{1}{2}$ are horizontal asymptotes.

d.