4.1 Group Activity Problems

PROCEDURE **Locating Absolute Extreme Values on a Closed Interval**

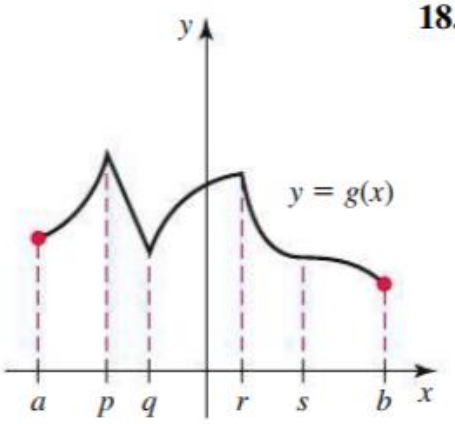
Assume the function f is continuous on the closed interval $[a, b]$.

1. Locate the critical points c in (a, b), where $f'(c) = 0$ or $f'(c)$ does not exist. These points are candidates for absolute maxima and minima.
2. Evaluate f at the critical points and at the endpoints of $[a, b]$.
3. Choose the largest and smallest values of f from Step 2 for the absolute maximum and minimum values, respectively.

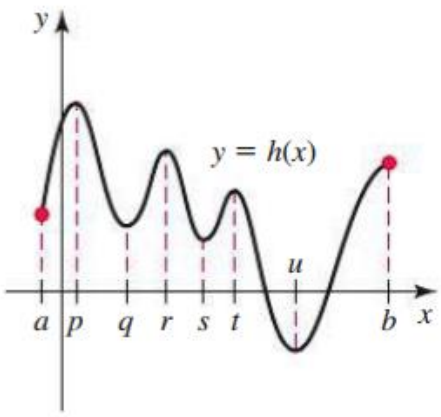
T/F Critical points may occur at the endpoints.

15–18. *Use the following graphs to identify the points on the interval $[a, b]$ at which local and absolute extreme values occur.*

17.



18.



19–22. *Sketch the graph of a continuous function f on $[0, 4]$ satisfying the given properties.*

22. $f'(x) = 0$ at $x = 1$ and 3; $f'(2)$ is undefined; f has an absolute maximum at $x = 2$; f has neither a local maximum nor a local minimum at $x = 1$; and f has an absolute minimum at $x = 3$.

23–42. Locating critical points *Find the critical points of the following functions. Assume a is a nonzero constant.*

36. $f(t) = t^2 - 2\ln(t^2 + 1)$

40. $f(x) = \dfrac{x}{\sqrt{x - a}}$

43–68. Absolute maxima and minima *Determine the location and value of the absolute extreme values of f on the given interval, if they exist.*

60. $f(x) = 2x^6 - 15x^4 + 24x^2$ on $[-2, 2]$

73. Trajectory high point A stone is launched vertically upward from a cliff 192 ft above the ground at a speed of 64 ft/s. Its height above the ground t seconds after the launch is given by $s = -16t^2 + 64t + 192$, for $0 \le t \le 6$. When does the stone reach its maximum height?

77. Explain why or why not Determine whether the following statements are true and give an explanation or counterexample.

a. The function $f(x) = \sqrt{x}$ has a local maximum on the interval $[0, \infty)$.

b. If a function has an absolute maximum on a closed interval, then the function must be continuous on that interval.

c. A function f has the property that $f'(2) = 0$. Therefore, f has a local extreme value at $x = 2$.

d. Absolute extreme values of a function on a closed interval always occur at a critical point or an endpoint of the interval.