## 4.3 Group Activity Problems


### Recap of Derivative Properties

This section has demonstrated that the first and second derivatives of a function provide valuable information about its graph. The relationships among a function's derivatives and its extreme values and concavity are summarized in Figure 4.43.

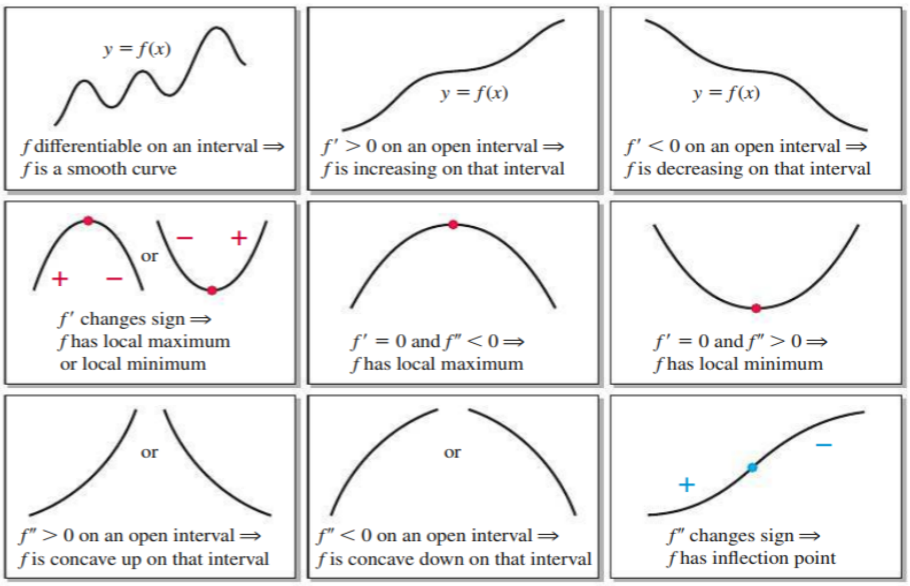



**Figure 4.43**

## Recitation Warm-Up / Poll Question

**63–76. Concavity** *Determine the intervals on which the following functions are concave up or concave down. Identify any inflection points.*

**64.** $f(x) = -x^4 - 2x^3 + 12x^2$

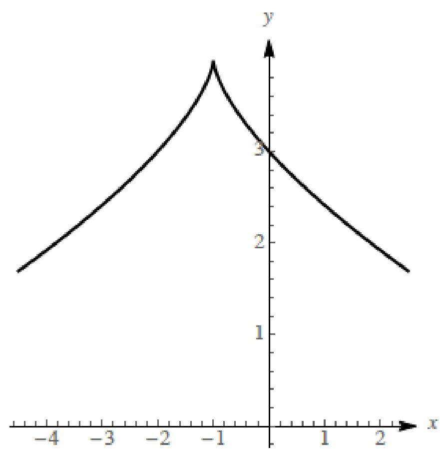
**4.3.64** $f'(x) = -4x^3 - 6x^2 + 24x$, so $f''(x) = -12x^2 - 12x + 24 = -12(x^2 + x - 2) = -12(x+2)(x-1)$. Note the $f''$ is zero for $x = -2$ and $x = 1$, so these are potential inflection points. Now note that $f''(-3) < 0$, $f''(0) > 0$, and $f''(2) < 0$. Thus $f$ is concave up on $(-2, 1)$ and is concave down on $(-\infty, -2)$ and on $(1, \infty)$. There are inflection points at $(-2, 48)$ and $(1, 9)$.

**9–12.** *Sketch a graph of a function $f$ that is continuous on $(-\infty, \infty)$ and has the following properties. Use a sign graph to summarize information about the function.*

**10.** $f'(-1)$ is undefined; $f'(x) > 0$ on $(-\infty, -1)$; $f'(x) < 0$ on $(-1, \infty)$

4.3.10

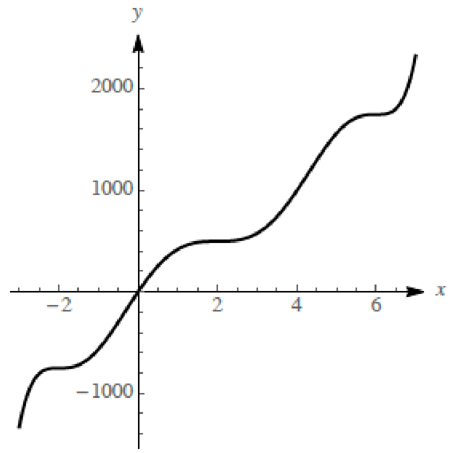
Such a function would be increasing on $(-\infty, -1)$, and decreasing on $(-1, \infty)$. It should have a point of non-differentiability at $x = -1$.




**12.** $f'(-2) = f'(2) = f'(6) = 0$; $f'(x) \geq 0$ on $(-\infty, \infty)$

4.3.12

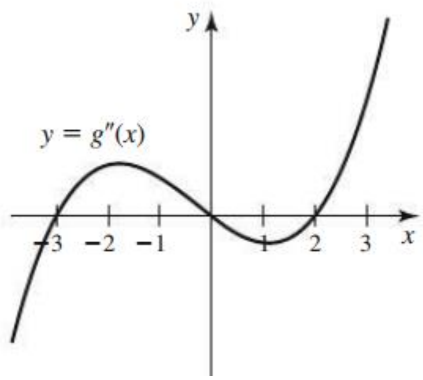
Such a function is never decreasing, but is flat at $-2$, 2, and 6.




**14.** The following graph of $g''$ has exactly three $x$-intercepts.

**a.** For what values of $x$ in $(-4, 3)$ is the graph of $g$ concave up? Concave down?

**b.** State the inflection points of $g$ that lie in $(-4, 3)$.




4.3.14

a. $g'' > 0$ on $(-3, 0)$ and $(2, 3)$, so $g$ is concave up on those intervals. $g'' < 0$ on $(-4, 3)$ and $(0, 2)$, so $g$ is concave down on those intervals.

b. There are inflection points at $x = -3$, $x = 0$, and $x = 2$.

**19–44. Increasing and decreasing functions** *Find the intervals on which $f$ is increasing and the intervals on which it is decreasing.*

**36.** $f(x) = x^2\sqrt{9 - x^2}$ on $(-3, 3)$

**4.3.36** $f'(x) = 2x\sqrt{9-x^2} + x^2 \cdot \frac{1}{2\sqrt{9-x^2}} \cdot (-2x) = \frac{2x(9-x^2)}{\sqrt{9-x^2}} + -\frac{x^3}{\sqrt{9-x^2}} = \frac{18x - 3x^3}{\sqrt{9-x^2}} = \frac{3x(6-x^2)}{\sqrt{9-x^2}}$. This is zero when $x = 0$, and when $x = \pm\sqrt{6}$. Note that $\sqrt{6} \approx 2.4$. Also note that $f'(-2.7) > 0$, $f'(-1) < 0$, $f'(1) > 0$ and $f'(2.7) < 0$. Thus, $f$ is increasing on $(-3, -\sqrt{6})$ and on $(0, \sqrt{6})$, and is decreasing on $(-\sqrt{6}, 0)$ and on $(\sqrt{6}, 3)$.

## 45–54. First Derivative Test

***a.*** *Locate the critical points of $f$.*

***b.*** *Use the First Derivative Test to locate the local maximum and minimum values.*

***c.*** *Identify the absolute maximum and minimum values of the function on the given interval (when they exist).*

**51.** $f(x) = x^{2/3}(x - 5)$ on $[-5, 5]$

**4.3.51**

a. $f'(x) = x^{2/3} + (x-5) \cdot \frac{2}{3}x^{-1/3} = \frac{5x - 10}{3x^{1/3}}$, which is undefined at $x = 0$ and is 0 at $x = 2$. So these are the two critical points.

b. Note that $f'(-1) > 0$ and $f'(1) < 0$, and $f'(3) > 0$ so $f$ has a local maximum at $x = 0$ of $f(0) = 0$ and a local minimum at $x = 2$ of $-3\sqrt[3]{4} \approx -4.762$.

c. Note that $f(-5) = -10\sqrt[3]{25} \approx -29.24$, $f(0) = 0$, and $f(5) = 0$, so the absolute maximum of $f$ on $[-5, 5]$ is 0 and the absolute minimum is $-10\sqrt[3]{25}$.

**63–76. Concavity** *Determine the intervals on which the following functions are concave up or concave down. Identify any inflection points.*

**73.** $f(x) = \sqrt{x}\ln x$

**4.3.73** $f'(x) = \sqrt{x}/x + (\ln x)\left(\frac{1}{2\sqrt{x}}\right) = \frac{2+\ln x}{2\sqrt{x}}$. $f''(x) = \frac{2\sqrt{x}/x - (2+\ln x)/\sqrt{x}}{(2\sqrt{x})^2} = -\frac{\ln x}{4\sqrt{x^3}}$. Note that $f''$ is 0 only at $x = 1$. On $(0, 1)$ we note that $f'' > 0$ so $f$ is concave up, and on $(1, \infty)$ we note that $f'' < 0$ so $f$ is concave down. There is an inflection point at $(1, 0)$.

**77–94. Second Derivative Test** *Locate the critical points of the following functions. Then use the Second Derivative Test to determine (if possible) whether they correspond to local maxima or local minima.*

**91.** $h(x) = (x + a)^4$; $a$ constant

**4.3.91** $f'(x) = 4(x + a)^3$, which is 0 for $x = -a$. Note that $f''(x) = 12(x + a)^2$, which is 0 at $x = -a$, so the test is inconclusive. The first derivative test shows that there is a local minimum at $x = -a$.

**107. Interpreting the derivative** The graph of $f'$ on the interval $[-3, 2]$ is shown in the figure.

**a.** On what interval(s) is $f$ increasing? Decreasing?

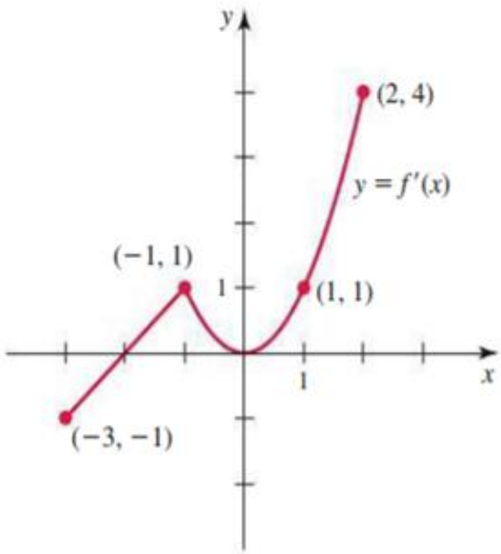
**b.** Find the critical points of $f$. Which critical points correspond to local maxima? Local minima? Neither?

**c.** At what point(s) does $f$ have an inflection point?

**d.** On what interval(s) is $f$ concave up? Concave down?

**e.** Sketch the graph of $f''$.

**f.** Sketch one possible graph of $f$.



**4.3.107**

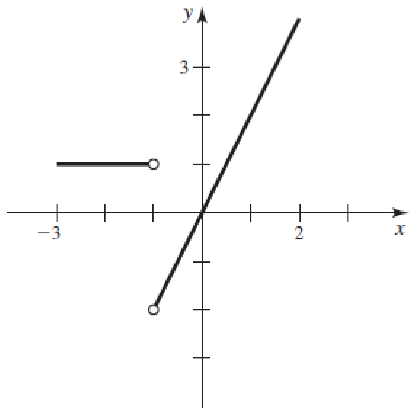
a. $f$ is increasing on $(-2, 2)$. It is decreasing on $(-3, -2)$.

b. There are critical points of $f$ at $x = -2$ and at $x = 0$. There is a local minimum at $x = -2$ and no extremum at $x = 0$.

c. There are inflection points of $f$ at $x = -1$ and at $x = 0$.

d. $f$ is concave up on $(-3, -1)$ and on $(0, 2)$, while it is concave down on $(-1, 0)$.

e.




f.