4.3 Group Activity Problems

Recap of Derivative Properties

This section has demonstrated that the first and second derivatives of a function provide valuable information about its graph. The relationships among a function's derivatives and its extreme values and concavity are summarized in Figure 4.43.

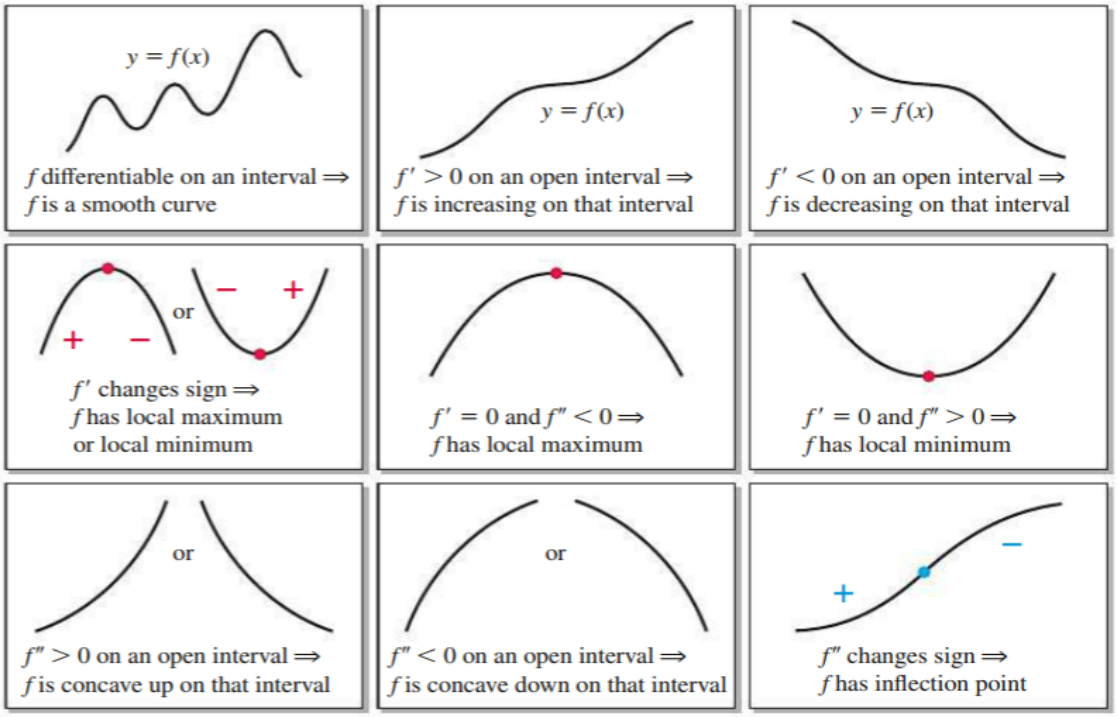


Figure 4.43

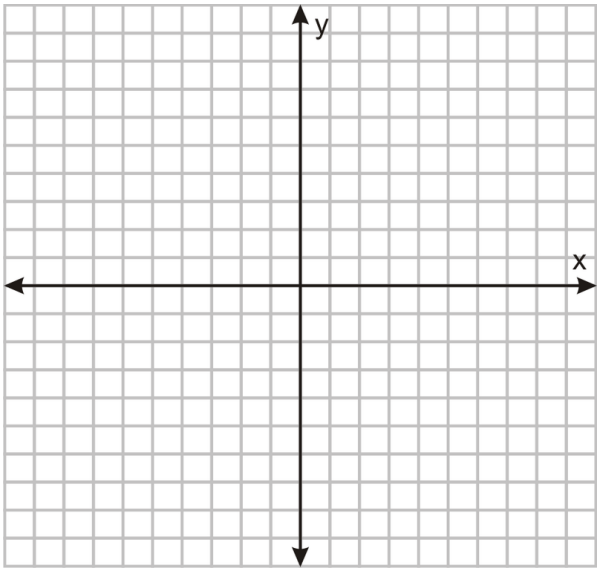
Recitation Warm-Up / Poll Question

63–76. Concavity *Determine the intervals on which the following functions are concave up or concave down. Identify any inflection points.*

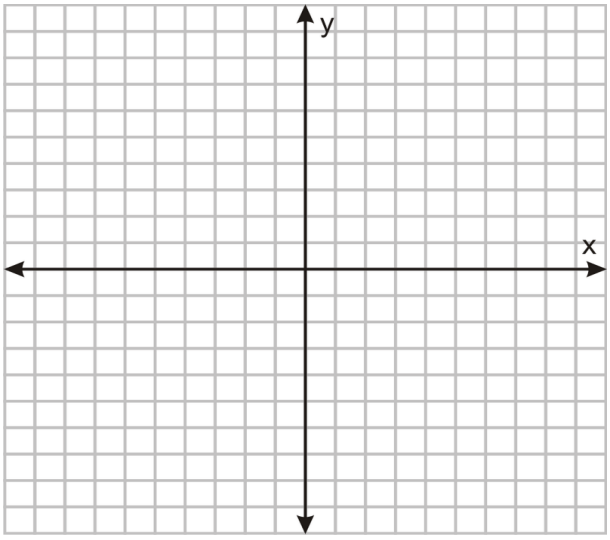
64. $f(x) = -x^4 - 2x^3 + 12x^2$

9–12. *Sketch a graph of a function f that is continuous on $(-\infty, \infty)$ and has the following properties. Use a sign graph to summarize information about the function.*

10. $f'(-1)$ is undefined; $f'(x) > 0$ on $(-\infty, -1)$; $f'(x) < 0$ on $(-1, \infty)$



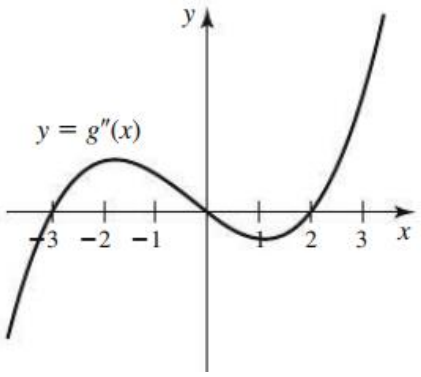
12. $f'(-2) = f'(2) = f'(6) = 0$; $f'(x) \geq 0$ on $(-\infty, \infty)$



14. The following graph of g'' has exactly three x-intercepts.

a. For what values of x in $(-4, 3)$ is the graph of g concave up? Concave down?

b. State the inflection points of g that lie in $(-4, 3)$.



19–44. Increasing and decreasing functions *Find the intervals on which f is increasing and the intervals on which it is decreasing.*

36. $f(x) = x^2\sqrt{9 - x^2}$ on $(-3, 3)$

45–54. First Derivative Test

a. Locate the critical points of f.
b. Use the First Derivative Test to locate the local maximum and minimum values.
c. Identify the absolute maximum and minimum values of the function on the given interval (when they exist).

51. $f(x) = x^{2/3}(x - 5)$ on $[-5, 5]$

63–76. Concavity *Determine the intervals on which the following functions are concave up or concave down. Identify any inflection points.*

73. $f(x) = \sqrt{x} \ln x$

77–94. Second Derivative Test *Locate the critical points of the following functions. Then use the Second Derivative Test to determine (if possible) whether they correspond to local maxima or local minima.*

91. $h(x) = (x + a)^4$; a constant

107. Interpreting the derivative The graph of f' on the interval $[-3, 2]$ is shown in the figure.

a. On what interval(s) is f increasing? Decreasing?
b. Find the critical points of f. Which critical points correspond to local maxima? Local minima? Neither?
c. At what point(s) does f have an inflection point?
d. On what interval(s) is f concave up? Concave down?
e. Sketch the graph of f''.
f. Sketch one possible graph of f.