4.7
~~4.1~~ Group Activity Problems

THEOREM 4.12 L'Hôpital's Rule
Suppose f and g are differentiable on an open interval I containing a with $g'(x) \neq 0$ on I when $x \neq a$. If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$, then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$

provided the limit on the right exists (or is $\pm\infty$). The rule also applies if $x \to a$ is replaced with $x \to \pm\infty$, $x \to a^+$, or $x \to a^-$.

THEOREM 4.13 L'Hôpital's Rule (∞/∞)
Suppose f and g are differentiable on an open interval I containing a, with $g'(x) \neq 0$ on I when $x \neq a$. If $\lim_{x\to a} f(x) = \pm\infty$ and $\lim_{x\to a} g(x) = \pm\infty$, then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$

provided the limit on the right exists (or is $\pm\infty$). The rule also applies for $x \to \pm\infty$, $x \to a^+$, or $x \to a^-$.

(Other/Secondary) Indeterminate Forms
$1^\infty, 0^0, \infty^0$ $\infty - \infty$ $0\cdot\infty$
use ln properties

Procedure:
1) "DSP" to obtain an indeterminate form
2) Re-write "other indeterminate forms" into "primary indeterminate form"
3) Use L.R.

6. Which of the following limits can be evaluated without l'Hôpital's Rule? Evaluate each limit.

a. $\lim_{x\to 0} \frac{\sin x}{x^3 + 2x + 1}$ **b.** $\lim_{x\to 0} \frac{\sin x}{x^3 + 2x}$

17–83. Limits *Evaluate the following limits. Use l'Hôpital's Rule when it is convenient and applicable.*

22. $\lim_{x\to 0} \frac{e^x - 1}{x^2 + 3x}$

24. $\lim_{x\to\infty} \frac{4x^3 - 2x^2 + 6}{\pi x^3 + 4}$

44. $\lim_{x \to 1} \dfrac{x^n - 1}{x - 1}$, n is a positive integer

51. $\lim_{x \to \infty} \dfrac{x^2 - \ln(2/x)}{3x^2 + 2x}$

52. $\lim_{x \to 1^+} \left(\dfrac{1}{x - 1} - \dfrac{1}{\sqrt{x - 1}} \right)$

80. $\lim_{x \to \infty} \left(1 + \dfrac{a}{x} \right)^x$, for a constant a

81. $\lim_{x \to 0} (e^{ax} + x)^{1/x}$, for a constant a

93. $\lim_{x \to 0} \dfrac{a^x - b^x}{x}$, for positive constants a and b Hint: (a^x)' = a^x * lna

94. $\lim_{x \to 0} (1 + ax)^{b/x}$, for positive constants a and b

105. Explain why or why not Determine whether the following statements are true and give an explanation or counterexample.

a. By l'Hôpital's Rule, $\lim_{x\to 2} \dfrac{x-2}{x^2-1} = \lim_{x\to 2} \dfrac{1}{2x} = \dfrac{1}{4}$.

b. $\lim_{x\to 0} x \sin x = \lim_{x\to 0} f(x)g(x) = \lim_{x\to 0} f'(x) \lim_{x\to 0} g'(x) = (\lim_{x\to 0} 1)(\lim_{x\to 0} \cos x) = 1$.

c. $\lim_{x\to 0^+} x^{1/x}$ is an indeterminate form.