4.9 Group Activity Problems - Solutions

DEFINITION Antiderivative

A function F is an **antiderivative** of f on an interval I provided $F'(x) = f(x)$, for all x in I.

THEOREM 4.15 The Family of Antiderivatives

Let F be any antiderivative of f on an interval I. Then *all* the antiderivatives of f on I have the form $F + C$, where C is an arbitrary constant.

69–76. Particular antiderivatives *For the following functions f, find the antiderivative F that satisfies the given condition.*

75. $f(y) = \dfrac{3y^3 + 5}{y}$; $F(1) = 3, y > 0$

76. $f(\theta) = 2\sin\theta - 4\cos\theta$; $F\left(\dfrac{\pi}{4}\right) = 2$

4.9.75 We have $F(y) = \int \frac{3y^3+5}{y}\,dy = \int \left(\frac{3y^3}{y} + \frac{5}{y}\right) dy = \int \left(3y^2 + \frac{5}{y}\right) dy = y^3 + 5\ln|y| + C$; substituting $F(1) = 3$ gives $1 + 0 + C = 3$, so $C = 2$, and thus $F(y) = y^3 + 5\ln y + 2$, $y > 0$.

4.9.76 $F(\theta) = \int (2\sin\theta - 4\cos\theta)\,d\theta = -2\cos\theta - 4\sin\theta + C$; substituting $F(\pi/4) = 2$ gives $-\sqrt{2} - 2\sqrt{2} + C = 2$, so $C = 2 + 3\sqrt{2}$, and thus $F(\theta) = -2\cos\theta - 4\sin\theta + 2 + 3\sqrt{2}$.

77–86. Solving initial value problems *Find the solution of the following initial value problems.*

82. $p'(t) = 10e^t + 70;\ p(0) = 100$

84. $u'(x) = \dfrac{xe^{2x} + 4e^x}{xe^x};\ u(1) = 0, x > 0$

4.9.82 $p(t) = \int (10e^t + 70)\, dt = 10e^t + 70t + C$; substituting $p(0) = 100$ gives $10 + C = 100$, so $C = 90$, and thus $p(t) = 10e^t + 70t + 90$.

4.9.84 $u(x) = \int \frac{xe^{2x} + 4e^x}{xe^x}\, dx = \int \left(e^x + \frac{4}{x}\right) dx = e^x + 4\ln|x| + C$; substituting $u(1) = 0$ gives $e + C = 0$, so $C = -e$, and thus $u(x) = e^x + 4\ln x - e$, $x > 0$.

111. Explain why or why not Determine whether the following statements are true and give an explanation or counterexample.

a. $F(x) = x^3 - 4x + 100$ and $G(x) = x^3 - 4x - 100$ are antiderivatives of the same function.

b. If $F'(x) = f(x)$, then f is an antiderivative of F.

c. If $F'(x) = f(x)$, then $\int f(x)\,dx = F(x) + C$.

d. $f(x) = x^3 + 3$ and $g(x) = x^3 - 4$ are derivatives of the same function.

e. If $F'(x) = G'(x)$, then $F(x) = G(x)$.

4.9.111

a. True, because $F'(x) = G'(x)$.

b. False; f is the derivative of F.

c. True; $\int f(x)\,dx$ is the most general antiderivative of $f(x)$, which is $F(x) + C$.

d. False; a function cannot have more than one derivative.

e. False; one can only conclude that $F(x)$ and $G(x)$ differ by a constant.

117. Flow rate A large tank is filled with water when an outflow valve is opened at $t = 0$. Water flows out at a rate, in gal/min, given by $Q'(t) = 0.1(100 - t^2)$, for $0 \le t \le 10$.

a. Find the amount of water $Q(t)$ that has flowed out of the tank after t minutes, given the initial condition $Q(0) = 0$.

b. Graph the flow function Q, for $0 \le t \le 10$.

c. How much water flows out of the tank in 10 min?

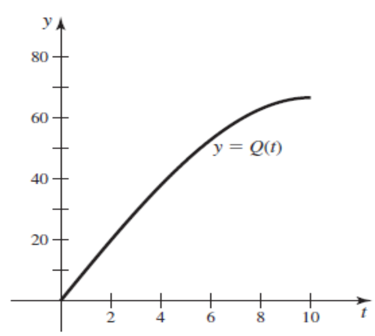
4.9.117

a. We have

$$Q(t) = \int 0.1(100 - t^2)\,dt = 0.1\left(100t - \frac{t^3}{3}\right) + C;$$

$Q(0) = 0$, so $C = 0$ and $Q(t) = 10t - t^3/30$ gal.

b.



c. $Q(10) = 200/3 \approx 67$ gal.