4.9 Group Activity Problems

DEFINITION Antiderivative

A function F is an **antiderivative** of f on an interval I provided $F'(x) = f(x)$, for all x in I.

THEOREM 4.15 The Family of Antiderivatives

Let F be any antiderivative of f on an interval I. Then *all* the antiderivatives of f on I have the form $F + C$, where C is an arbitrary constant.

69–76. Particular antiderivatives *For the following functions f, find the antiderivative F that satisfies the given condition.*

75. $f(y) = \dfrac{3y^3 + 5}{y}; F(1) = 3, y > 0$

76. $f(\theta) = 2\sin\theta - 4\cos\theta; F\left(\dfrac{\pi}{4}\right) = 2$

77–86. Solving initial value problems *Find the solution of the following initial value problems.*

82. $p'(t) = 10e^t + 70; p(0) = 100$

84. $u'(x) = \dfrac{xe^{2x} + 4e^x}{xe^x}; u(1) = 0, x > 0$

111. Explain why or why not Determine whether the following statements are true and give an explanation or counterexample.

a. $F(x) = x^3 - 4x + 100$ and $G(x) = x^3 - 4x - 100$ are antiderivatives of the same function.

b. If $F'(x) = f(x)$, then f is an antiderivative of F.

c. If $F'(x) = f(x)$, then $\int f(x)\,dx = F(x) + C$.

d. $f(x) = x^3 + 3$ and $g(x) = x^3 - 4$ are derivatives of the same function.

e. If $F'(x) = G'(x)$, then $F(x) = G(x)$.

117. Flow rate A large tank is filled with water when an outflow valve is opened at $t = 0$. Water flows out at a rate, in gal/min, given by $Q'(t) = 0.1(100 - t^2)$, for $0 \le t \le 10$.

a. Find the amount of water $Q(t)$ that has flowed out of the tank after t minutes, given the initial condition $Q(0) = 0$.

b. Graph the flow function Q, for $0 \le t \le 10$.

c. How much water flows out of the tank in 10 min?