5.3 Group Activity Problems - Solutions

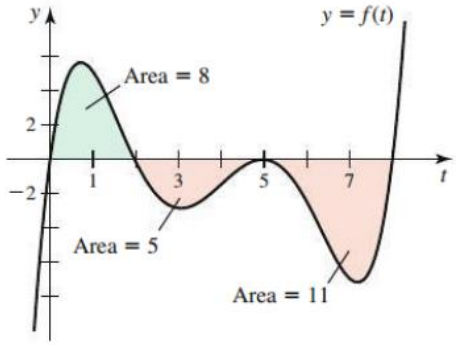
9. Evaluate $\dfrac{d}{dx}\displaystyle\int_a^x f(t)\,dt$ and $\dfrac{d}{dx}\displaystyle\int_a^b f(t)\,dt$, where a and b are constants.

5.3.9 $\dfrac{d}{dx}\displaystyle\int_a^x f(t)\,dt = f(x)$, and $\dfrac{d}{dx}\displaystyle\int_a^b f(t)\,dt = 0$. The latter is the derivative of a constant, the former follows from the Fundamental Theorem.

14. Area functions The graph of f is shown in the figure. Let $A(x) = \int_0^x f(t)\,dt$ and $F(x) = \int_2^x f(t)\,dt$ be two area functions for f. Evaluate the following area functions.

a. $A(2)$ **b.** $F(5)$ **c.** $A(0)$ **d.** $F(8)$

e. $A(8)$ **f.** $A(5)$ **g.** $F(2)$



5.3.14

a. $A(2) = \displaystyle\int_0^2 f(t)\,dt = 8.$

b. $F(5) = \displaystyle\int_2^5 f(t)\,dt = -5.$

c. $A(0) = \displaystyle\int_0^0 f(t)\,dt = 0.$

d. $F(8) = \displaystyle\int_2^8 f(t)\,dt = -16.$

e. $A(8) = \displaystyle\int_0^8 f(t)\,dt = 8 - 16 = -8.$

f. $A(5) = \displaystyle\int_0^5 f(t)\,dt = 8 - 5 = 3.$

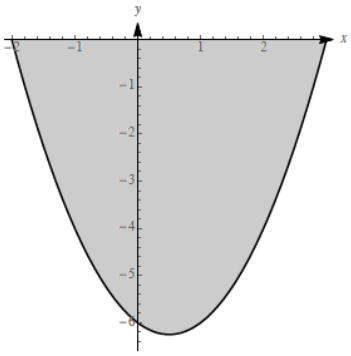
g. $F(2) = \displaystyle\int_2^2 f(t)\,dt = 0.$

25–28. Definite integrals *Evaluate the following integrals using the Fundamental Theorem of Calculus. Sketch the graph of the integrand and shade the region whose net area you have found.*

25. $\int_{-2}^{3}(x^2 - x - 6)\,dx$ **26.** $\int_{0}^{1}(x - \sqrt{x})\,dx$

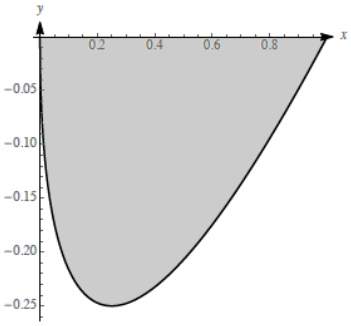
5.3.25

$$\int_{-2}^{3}(x^2 - x - 6)\,dx = \left(\frac{x^3}{3} - \frac{x^2}{2} - 6x\right)\Big|_{-2}^{3} = -\frac{125}{6}.$$



5.3.26

$$\int_{0}^{1}(x-\sqrt{x})\,dx = \left(\frac{x^2}{2} - \frac{2}{3}x^{3/2}\right)\Big|_{0}^{1} = \frac{1}{2} - \frac{2}{3} - (0-0) = -\frac{1}{6}.$$



73–86. Derivatives of integrals *Simplify the following expressions.*

76. $\dfrac{d}{dx}\displaystyle\int_x^0 \frac{dp}{p^2+1}$

78. $\dfrac{d}{dx}\displaystyle\int_0^{x^2} \frac{dt}{t^2+4}$

5.3.76 This is $-\dfrac{d}{dx}\displaystyle\int_0^x \frac{dp}{p^2+1} = \frac{-1}{x^2+1}$.

5.3.78 $\dfrac{d}{dx}\displaystyle\int_0^{x^2} \frac{1}{t^2+4}\,dt = \frac{2x}{x^4+4}$.