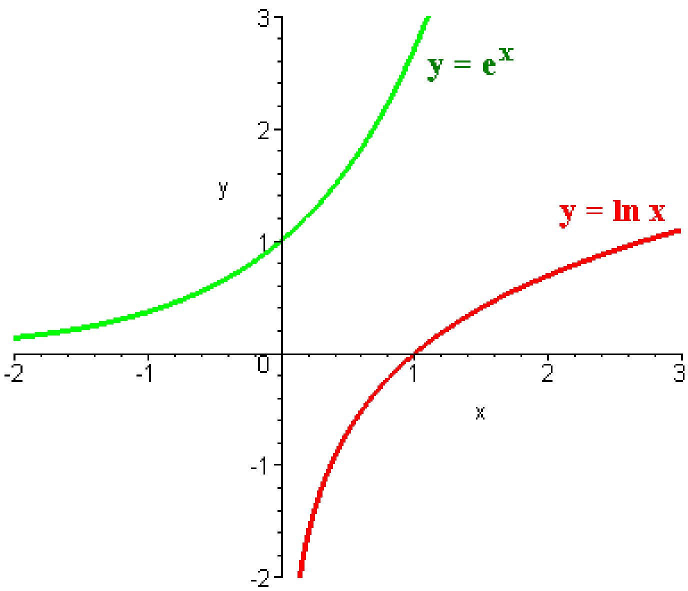
Precalculus Review - Group Activity Problems

Solutions

1.3 Exponential, Logarithmic Functions Review



Review: properties of ln

1) $\ln(ab) = \ln a + \ln b$
2) $\ln \frac{a}{b} = \ln a - \ln b$
3) $\ln a^k = k \ln a$
4) $\ln e = 1$
5) $\ln 1 = 0$

19. Evaluate each expression without a calculator.

a. $\log_{10} 1000$ **b.** $\log_2 16$ **c.** $\log_{10} 0.01$ **d.** $\ln e^3$ **e.** $\ln \sqrt{e}$

1.3.19

a. Because $10^3 = 1000$, $\log_{10} 1000 = 3$.

b. Because $2^4 = 16$, $\log_2 16 = 4$.

c. Because $10^{-2} = \frac{1}{100} = 0.01$, $\log_{10} 0.01 = -2$.

d. Because e^x and $\ln x$ are inverses, $\ln e^3 = 3$.

e. Because e^x and $\ln x$ are inverses, $\ln \sqrt{e} = \ln e^{1/2} = \frac{1}{2}$.

45–50. Properties of logarithms *Assume* $\log_b x = 0.36$, $\log_b y = 0.56$, *and* $\log_b z = 0.83$. *Evaluate the following expressions.*

45. $\log_b \frac{x}{y}$

46. $\log_b x^2$

47. $\log_b xz$

48. $\log_b \frac{\sqrt{xy}}{z}$

1.3.45 $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y = 0.36 - .056 = -0.2.$

1.3.46 $\log_b x^2 = 2\log_b x = 2(0.36) = 0.72.$

1.3.47 $\log_b xz = \log_b x + \log_b z = 0.36 + 0.83 = 1.19.$

1.3.48 $\log_b \frac{\sqrt{xy}}{z} = \log_b(xy)^{1/2} - \log_b z = \frac{1}{2}(\log_b x + \log_b y) - \log_b z = (0.36)/2 + (0.56)/2 - 0.83 = -0.37.$

51–60. Solving equations *Solve the following equations.*

59. $3^{3x-4} = 15$

60. $5^{3x} = 29$

1.3.59 Since $3^{3x-4} = 15$, we have that $\ln 3^{3x-4} = \ln 15$, so $(3x-4)\ln 3 = \ln 15$. Thus, $3x - 4 = \frac{\ln 15}{\ln 3}$, so $x = \frac{(\ln 15)/(\ln 3)+4}{3} = \frac{\ln 15 + 4\ln 3}{3\ln 3} = \frac{\ln 5 + \ln 3 + 4\ln 3}{3\ln 3} = \frac{\ln 5}{3\ln 3} + \frac{5}{3}$.

1.3.60 Since $5^{3x} = 29$, we have that $\ln 5^{3x} = \ln 29$, so $(3x)\ln 5 = \ln 29$. Solving for x gives $x = \frac{\ln 29}{3\ln 5}$.

77. Explain why or why not Determine whether the following statements are true and give an explanation or counterexample.

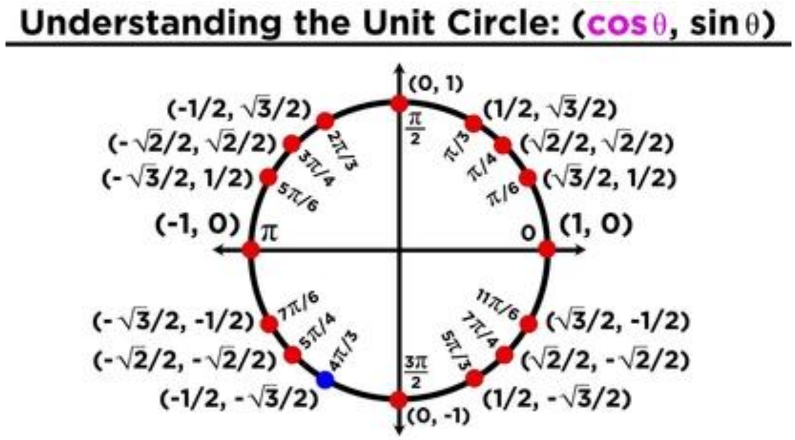
a. If $y = 3^x$, then $x = \sqrt[3]{y}$.

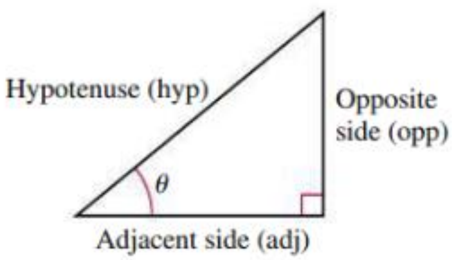
b. $\frac{\log_b x}{\log_b y} = \log_b x - \log_b y$

1.3.77

a. False. For example, $3 = 3^1$, but $1 \neq \sqrt[3]{3}$.

b. False. For example, suppose $x = y = b = 2$. Then the left-hand side of the equation is equal to 1, but the right-hand side is 0.



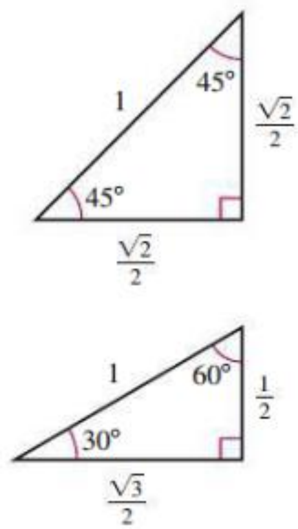


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \quad \cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} \quad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

Standard triangles



Trigonometric Identities

Reciprocal Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \cot^2\theta = \csc^2\theta \qquad \tan^2\theta + 1 = \sec^2\theta$$

Solve the following equations:

36. $2\theta \cos \theta + \theta = 0$

38. $\cos^2 \theta = \frac{1}{2}, 0 \leq \theta < 2\pi$

1.4.36 Given that $2\theta\cos(\theta) + \theta = 0$, we have $\theta(2\cos(\theta) + 1) = 0$. Which means that either $\theta = 0$, or $2\cos(\theta) + 1 = 0$. The latter leads to the equation $\cos\theta = -1/2$, which occurs at $\theta = 2\pi/3$ and $\theta = 4\pi/3$.

1.4.38 Given that $\cos^2\theta = \frac{1}{2}$, we have $|\cos\theta| = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$. Thus $\cos\theta = \frac{\sqrt{2}}{2}$ or $\cos\theta = -\frac{\sqrt{2}}{2}$. We have $\theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$.

91. Explain why or why not Determine whether the following statements are true and give an explanation or counterexample.

a. $\sin(a + b) = \sin a + \sin b$

b. The equation $\cos \theta = 2$ has multiple solutions.

1.4.91

a. False. For example, $\sin(\pi/2 + \pi/2) = \sin(\pi) = 0 \neq \sin(\pi/2) + \sin(\pi/2) = 1 + 1 = 2$.

b. False. That equation has zero solutions, because the range of the cosine function is $[-1, 1]$.

92–95. One function gives all six *Given the following information about one trigonometric function, evaluate the other five functions.*

92. $\sin \theta = -\frac{4}{5}$ and $\pi < \theta < \frac{3\pi}{2}$

94. $\sec \theta = \frac{5}{3}$ and $\frac{3\pi}{2} < \theta < 2\pi$

1.4.92 If $\sin\theta = -4/5$, then the Pythagorean identity gives $|\cos\theta| = 3/5$. But if $\pi < \theta < 3\pi/2$, then the cosine of θ is negative, so $\cos\theta = -3/5$. Thus $\tan\theta = 4/3$, $\cot\theta = 3/4$, $\sec\theta = -5/3$, and $\csc\theta = -5/4$.

1.4.94 If $\sec\theta = 5/3$, then $\cos\theta = 3/5$, and the Pythagorean identity gives $|\sin\theta| = 4/5$. But if $3\pi/2 < \theta < 2\pi$, then the sine of θ is negative, so $\sin\theta = -4/5$. Thus $\tan\theta = -4/3$, $\cot\theta = -3/4$, and $\csc\theta = -5/4$.