Chapter 5.2-5.3 Worksheet Problems – Solutions

51. Properties of integrals Use only the fact that $\int_0^4 3x(4-x)\,dx = 32$, and the definitions and properties of integrals, to evaluate the following integrals, if possible.

a. $\int_4^0 3x(4-x)\,dx$ **b.** $\int_0^4 x(x-4)\,dx$

c. $\int_4^0 6x(4-x)\,dx$ **d.** $\int_0^8 3x(4-x)\,dx$

5.2.51

a. $\int_4^0 3x(4-x)\,dx = -\int_0^4 3x(4-x)\,dx = -32.$

b. $\int_0^4 x(x-4)\,dx = -\frac{1}{3}\int_0^4 3x(4-x)\,dx = -\frac{1}{3}\cdot 32 = -\frac{32}{3}.$

c. $\int_4^0 6x(4-x)\,dx = -2\cdot\int_0^4 3x(4-x)\,dx = -2\cdot 32 = -64.$

d. $\int_0^8 3x(4-x)\,dx = \int_0^4 3x(4-x)\,dx + \int_4^8 3x(4-x)\,dx = 32 + \int_4^8 3x(4-x)\,dx$. It is not possible to evaluate the given integral from the information given.

Evaluate the following by using FTC:

49. $\int_1^8 \sqrt[3]{y}\, dy$

50. $\frac{1}{2}\int_0^{\ln 2} e^x\, dx$

51. $\int_1^4 \frac{x-2}{\sqrt{x}}\, dx$

52. $\int_1^2 \frac{2s^2-4}{s^3}\, ds$

5.3.49 $\int_1^8 \sqrt[3]{y}\, dy = \frac{3}{4}y^{4/3}\Big|_1^8 = 12 - \frac{3}{4} = \frac{45}{4}.$

5.3.50 $\frac{1}{2}\int_0^{\ln 2} e^x\, dx = \frac{1}{2}\left(e^x\Big|_0^{\ln 2}\right) = \frac{1}{2}(2-1) = \frac{1}{2}.$

5.3.51

$$\int_1^4 \frac{x-2}{\sqrt{x}}\, dx = \int_1^4 \left(\frac{x}{\sqrt{x}} - \frac{2}{\sqrt{x}}\right) dx = \int_1^4 \left(x^{1/2} - 2x^{-1/2}\right) dx$$
$$= \left(\frac{2}{3}x^{3/2} - 4x^{1/2}\right)\Big|_1^4 = \frac{16}{3} - 8 - \left(\frac{2}{3} - 4\right) = \frac{14}{3} - \frac{12}{3} = \frac{2}{3}.$$

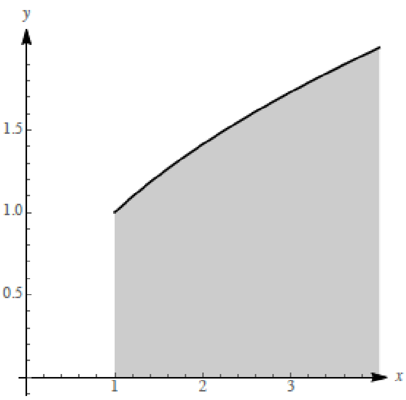
5.3.52 $\int_1^2 \left(\frac{2}{s} - \frac{4}{s^3}\right) ds = \left(2\ln|s| + \frac{2}{s^2}\right)\Big|_1^2 = 2\ln 2 + \frac{1}{2} - (0+2) = \ln 4 - \frac{3}{2}.$

63–66. Area *Find (i) the net area and (ii) the area of the following regions. Graph the function and indicate the region in question.*

63. The region bounded by $y = x^{1/2}$ and the x-axis between $x = 1$ and $x = 4$

5.3.63

The area (and net area) of this region is given by $\int_1^4 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_1^4 = \frac{16}{3} - \frac{2}{3} = \frac{14}{3}$.



Evaluate the definite integrals.

69. $\int_0^2 x^3\sqrt{16 - x^4}\,dx$

70. $\int_{-1}^1 (x - 1)(x^2 - 2x)^7\,dx$

5.5.69 Let $u = 16 - x^4$. Then $du = -4x^3\,dx$. Also note that when $x = 0$ we have $u = 16$, and when $x = 2$ we have $u = 0$. Substituting yields $\frac{1}{4}\int_0^{16} \sqrt{u}\,du = \frac{1}{4}\left(\frac{2u^{3/2}}{3}\right)\Big|_0^{16} = \frac{32}{3}$.

5.5.70 Let $u = x^2 - 2x$. Then $du = 2(x - 1)\,dx$. Also note that when $x = -1$ we have $u = 3$ and when $x = 1$ we have $u = -1$. Substituting yields $\frac{1}{2}\int_3^{-1} u^7\,du = \frac{1}{16}\left(u^8\right)\Big|_3^{-1} = \frac{1}{16}\left(1 - 3^8\right) = -\frac{6560}{16} = -410$.