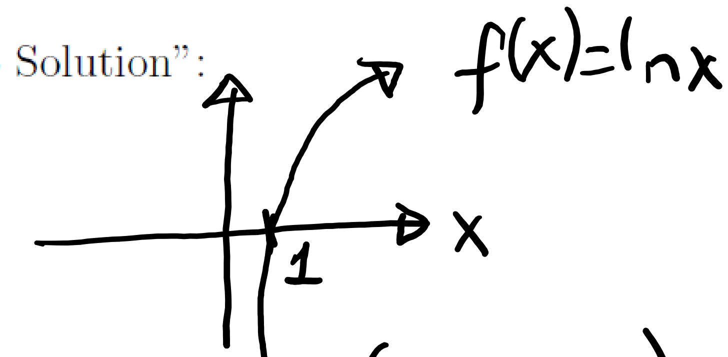
Midterm#1 Review

Monday, February 8, 2021 8:34 AM

$$\ln x - \ln y = \ln\left(\frac{x}{y}\right)$$

1. Find all solutions to the following equation. If there is no solution, write "No Solution":

$$\underbrace{\ln\left(\frac{2x^2}{3-x}\right) - \ln(x)}_{\text{Combine}} = \ln\left(\frac{4x}{4-x}\right)$$



$D: (0, \infty)$

$$\cancel{\ln}\left(\frac{\frac{2x^2}{3-x}}{\frac{x}{1}}\right) = \cancel{\ln}\left(\frac{4x}{4-x}\right)$$

$$\frac{2x^{\cancel{2}}}{3-x} \cdot \frac{1}{\cancel{x}} = \frac{4x}{4-x} \Rightarrow \frac{2x}{3-x} = \frac{4x}{4-x}$$

$$2x(4-x) = 4x(3-x) \Rightarrow 8x - 2x^2 = 12x - 4x^2$$

$$2x^2 = 4x$$

$$2x^2 - 4x = 0$$

$$2x(x-2) = 0$$

$$x = \cancel{0}, 2$$

$$\boxed{x = 2}$$

$\ln 0 \to$ undef.

$\ln 1 = 0$

verify: $\ln\left(\frac{2x^2}{3-x}\right) - \ln(x) = \ln\left(\frac{4x}{4-x}\right)$

$$\ln\left(\frac{2 \cdot 2^2}{3-2}\right) - \ln 2 = \ln\left(\frac{4 \cdot 2}{4-2}\right)$$

$$\underbrace{\ln 8 - \ln 2}_{\ln\left(\frac{8}{2}\right)} = \ln 4$$

$$= \ln 4 \quad \checkmark$$

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2. Solve the inequality $ax^2 + 4ax - 32a < 0$, a is an unknown positive constant. Write your answer using interval notation.

$$a(x^2+4x-32) < 0$$

$8 \quad -4$

$$\underbrace{a(x+8)(x-4)}_{+} < 0 \qquad -$$

$x=-8$, $x=4$

test P: $x = -10, 0, 10$

$$a\underbrace{(-10+8)}_{-}\underbrace{(-10-4)}_{-} \quad (+) \; -$$

$x=0$

$$a\underbrace{(0+8)}_{+}\underbrace{(0-4)}_{-} \rightarrow -$$

sign chart

$-\infty$	-10	-8	0	4	10	$+\infty$
$\underbrace{(x+8)\cdot(x-4)}$	$+$		$-$		$+$	
$a\cdot(\ldots)$	$-$		$+$		$-$	

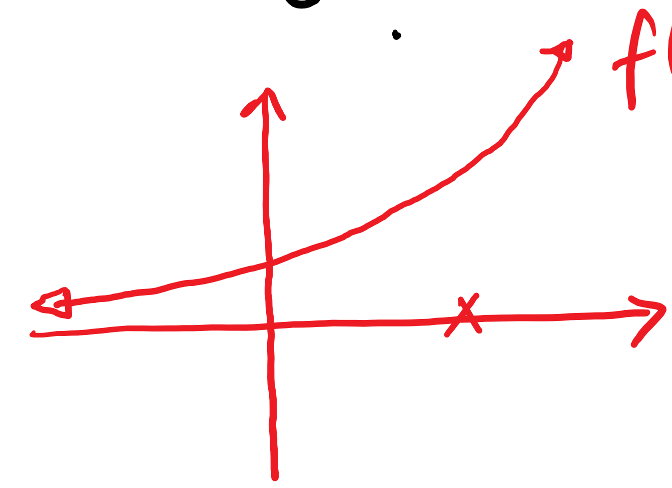
Solution $(-8, 4)$

if a is $(-)$: $(-\infty, -8), (4, \infty)$ $\cup$

3. The solution to the exponential equation: $12x^2 \cdot e^x = b \cdot x \cdot e^x$ is given as $1/6$. Find the value of the constant b, note that $x \neq 0$.

$x = 1/6$ $\qquad$ $b = ?$

$f(x) = e^x$



$$12x^2 \cdot e^x - b \cdot x \cdot e^x = 0$$

$$\underbrace{x}_{0} \cdot \underbrace{\cancel{e^x}}_{0} \boxed{\underbrace{(12x - b)}_{0}} = 0$$

$x \neq 0$, $e^x \neq 0$

$$12x - b = 0 \Rightarrow x = \frac{b}{12} = \frac{1}{6}$$

$$\boxed{b = 2}$$

4. Let $f(x) = 11x^2 - 11x + 6$. Simplify the difference quotient $\dfrac{f(10+h) - f(10)}{h}$ as much as possible (Assume $h \neq 0$). .

$f(x) = 11x^2 - 11x + 6$

$f(10+h) = 11(10+h)^2 - 11(10+h) + 6$

$(10+h)(10+h)$

$10h + 10h = 20h$

$f(10) = 11 \cdot 10^2 - 11 \cdot 10 + 6$

$(10^2 + 20h + h^2)$

$$\frac{11(10+h)^2 - 11(10+h) + 6 - (11 \cdot 10^2 - 11 \cdot 10 + 6)}{h}$$

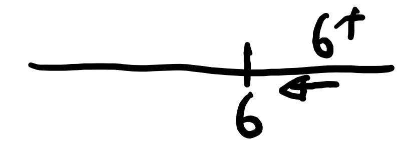
$11 \cdot 10^2 + 11 \cdot 20h + 11 \cdot h^2$

$$\frac{11(10^2 + 20h + h^2) - 11 \cdot 10 - 11 \cdot h + 6 - 11 \cdot 10^2 + 11 \cdot 10 - 6}{h}$$

$209h$

$$\frac{11 \cdot 20h + 11h^2 - 11h}{h} = \frac{\cancel{h}(209 + 11h)}{\cancel{h}} = \boxed{209 + 11h}$$

$11 \cdot 20h - 11h = 11h(20-1) = 11 \cdot 19h + 11h^2 = \dfrac{11h(19+h)}{h} = 11(19+h)$



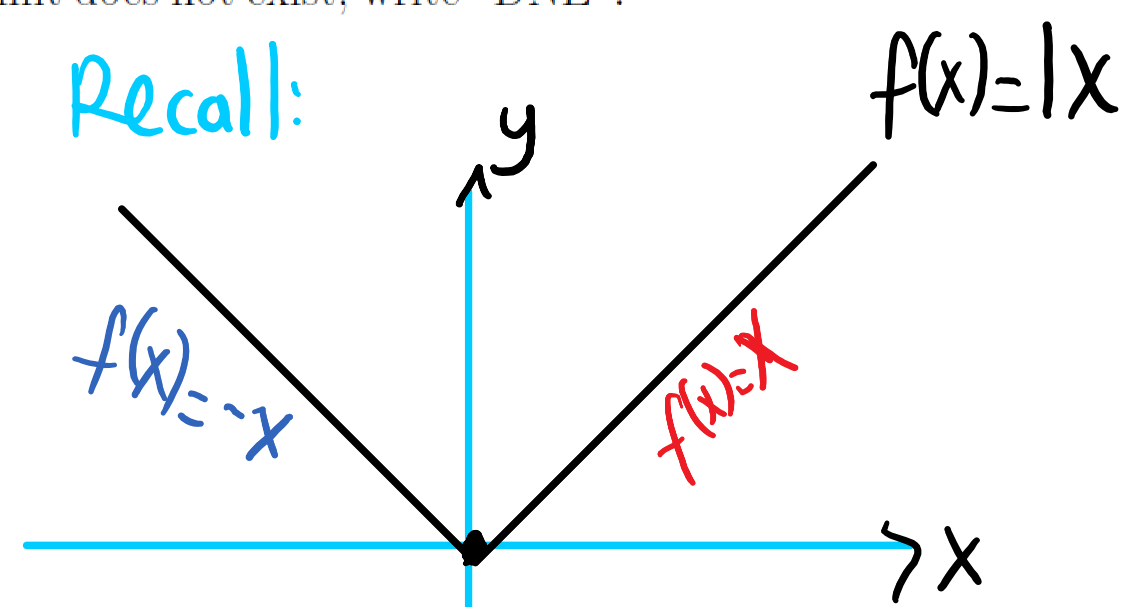
abs. value

5. Evaluate the limit or determine that it does not exist. If the limit does not exist, write "DNE".
$\lim_{x\to 6^+}\left(\frac{|36-x^2|}{x-6}\right)$.

$x \to 6^+$ $|36-x^2|$

$\underbrace{36-(6.000001)^2}_{<0}$

Recall:



$$f(x)=|x| \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\lim_{x\to 6^+}\left(\frac{\overbrace{|36-x^2|}^{<0}}{x-6}\right) \overset{DSP}{=} \text{"}\frac{0}{0}\text{"}$$

$-6+x = x-6$

$$\lim_{x\to 6^+}\left(\frac{-(36-x^2)}{x-6}\right) = \lim_{x\to 6^+}\left(\frac{-(6-x)\boxed{(6+x)}}{\cancel{x-6}}\right) = \lim_{x\to 6^+}(6+x) \overset{DSP}{=} 12$$

Important observations:

$$\frac{-2\cdot 3}{13} \neq \frac{-2\cdot -3}{13}$$

$$\underbrace{\frac{-2\cdot 3}{13} = \frac{2\cdot 3}{-13} = -\frac{2\cdot 3}{13}}_{\text{all same}}$$

Template $\lim_{x\to 0}\left(\frac{(kx)}{\sin(kx)}\right)=1$, $\lim_{x\to 0}\left(\frac{\sin(kx)}{(kx)}\right)=1$

6. Evaluate the limit or determine that it does not exist. If the limit does not exist, write "DNE".
$\lim_{x\to 0}\left(\frac{2x^2}{\sin^2(5x)}\right)$.

$$\lim_{x\to 0}\left(\frac{2x\cdot x}{\sin(5x)\cdot\sin(5x)}\right)\cdot\underbrace{\frac{5}{5}\cdot\frac{5}{5}}_{1}=\lim_{x\to 0}\left(\frac{2}{5\cdot 5}\cdot\frac{5x}{\sin(5x)}\cdot\frac{5x}{\sin(5x)}\right)$$

$1 \quad 1$

$$\lim_{x\to 0}\left(\frac{2}{5\cdot 5}\right)=\frac{2}{25}$$

7. Based on the given function below:

$$f(x)=\begin{cases} kx^3+e^{x-1} & , \ x<1 \\ 3x-\ln(2x-1) & , \ x\ge 1\end{cases}$$

(a) For what value of k (an unspecified constant) does $\lim_{x\to 1} f(x)$ exists. Explain.

(b) Given that $\underbrace{\lim_{x\to 1} f(x)}_{L}$ exists, what is its value?

LL = RL

a) $\left.\begin{array}{l}\lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)=L\end{array}\right\}$ $\lim_{x\to 1} f(x)=L$

$$\lim_{x\to 1^-}(kx^3+e^{x-1})=\lim_{x\to 1^+}(3x-\ln(2x-1))$$

$$k\cdot 1^3+e^{1-1}=3\cdot 1-\ln(2\cdot 1-1)$$

$$k+\cancel{e^0}^{\,1}=3-\cancel{\ln 1}^{\,0}$$

$$k+1=3$$

$$\boxed{k=2}$$

$e^0=1$
$\ln 1=0$
$\ln 0\to$ undef.

b) $\lim_{x\to 1} f(x)=\lim_{x\to 1^-}(2x^3+e^{x-1})$

$\overset{DSP}{=} 2\cdot 1^3+e^{1-1}$

$= 2+1=\boxed{3}$

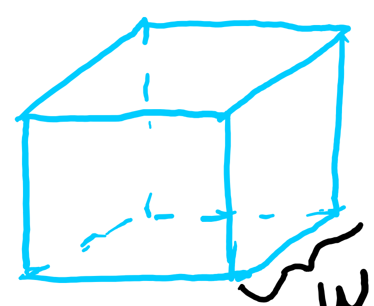
8. A rectangular box is constructed according to the following rules.

- the length of the box is twice its width
- the height of the box is 5 feet more than three times the length

(a) If w is the width of the box in feet, write an expression for $V(w)$, the volume of the box in cubic feet as a function of its width.

(b) Suppose the rules also require that the sum of the box's width and height to be no more than 26 feet. Under this condition, what is the domain of the function $V(w)$?

a)



$h = 3\cdot l + 5$

$= 3 \cdot 2w + 5$

$= 6w + 5$

$l = 2w$, w

$V_{\text{rect. prism}} = w \cdot l \cdot h$

$V(w) = w \cdot 2w \cdot (6w+5)$

$= 2w^2(6w+5)\ ft^3$

b) $w + h \leq 26$

$w + 6w + 5 \leq 26 \Rightarrow 7w + 5 \leq 26$

$7w \leq 21$

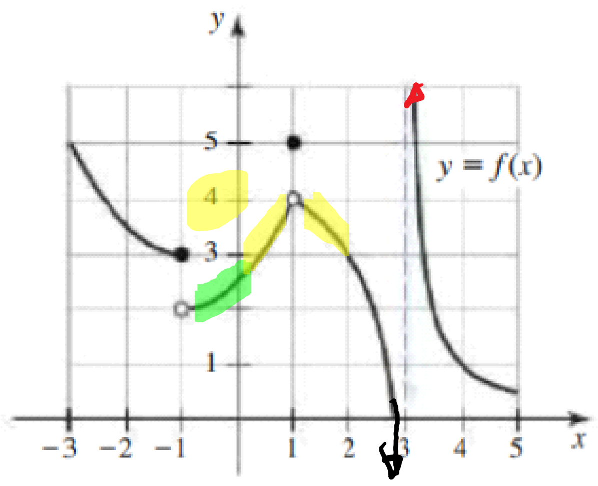
$w \leq 3$

Domain of $V(w)$: $(0, 3]$ interval notation

$l, w, h > 0$

$\cancel{l, w, h \geq 0}$ box to exist!

9. Use the graph of $f(x)$ below to find the following limits.



(a) $\lim_{x\to -1^-} f(x)$ = 3

(b) $\lim_{x\to -1^+} f(x)$ = 2

(c) $\lim_{x\to -1} f(x)$ DNE $2 \neq 3$

(d) $\lim_{x\to 1^-} f(x)$ = 4

(e) $\lim_{x\to 1^+} f(x)$ = 4

(f) $\lim_{x\to 1} f(x)$ = 4 } LL = RL

(g) $\lim_{x\to 3^-} f(x)$ = $-\infty$

(h) $\lim_{x\to 3^+} f(x)$ = $+\infty$

(i) $\lim_{x\to 3} f(x)$ DNE

$$\frac{\text{non-zero } \#}{\# \text{ app. } 0} \to \pm\infty$$

10. Let $f(x) = \dfrac{x^2 - 7x + 12}{x - a}$

(a) For what values of a, if any, does $\lim\limits_{x \to a^+} f(x)$ equals a finite number?

(b) For what values of a, if any, does $\lim\limits_{x \to a^+} f(x) = \infty$

(c) For what values of a, if any, does $\lim\limits_{x \to a^+} f(x) = -\infty$

a) $$\lim_{x \to a^+} \left(\frac{x^2 - 7x + 12}{x - a} \right) = \lim_{x \to a^+} \left(\frac{\cancel{(x-4)}\cancel{(x-3)}}{\underbrace{x - a}} \right) \to \text{finite } \#$$

$a^+ - a > 0$ app. 0

$\cancel{x-4}$ $\cancel{x-3}$

If $\boxed{a = 3 \text{ or } a = 4}$ we get a finite # as the limit

b) $$\lim_{x \to a^+} \left(\frac{(x-4)(x-3)}{\underbrace{x-a}_{+}} \right) = +\infty = \frac{\text{pos. non-zero } \#}{\text{pos. } \# \text{ app. } 0} = +\infty$$

$4, 3$

	$-\infty$	3	4	$+\infty$
	-10	3.5	10	
$(x-4)(x-3)$	$+$	$-$	$+$	

x is in $\boxed{(-\infty, 3) \cup (4, \infty)}$

c) x is in $\boxed{(3, 4)}$

11. Evaluate the limit or determine that it does not exist. If the limit does not exist, write "DNE". You must use proper calculus methods and notation to receive full credit.

$$\lim_{x\to 1} f(x), \text{ where } f(x) = \begin{cases} \dfrac{\sqrt{x}-1}{x-1} & , x > 1 \\ 8 & , x = 1 \\ \dfrac{2x-2}{x^2+2x-3} & , x < 1 \end{cases}$$

to exist: $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

$$\lim_{x\to 1^-}\left(\frac{2x-2}{(x+3)(x-1)}\right) \overset{DSP}{=} \text{"}\frac{0}{0}\text{"}$$

$$\lim_{x\to 1^-}\left(\frac{2\cancel{(x-1)}}{(x+3)\cancel{(x-1)}}\right) = \lim_{x\to 1^-}\left(\frac{2}{x+3}\right) = \frac{2}{4} = \frac{1}{2}$$

$$\lim_{x\to 1^+}\left(\frac{\sqrt{x}-1}{x-1}\right) \overset{DSP}{=} \text{"}\frac{0}{0}\text{"}$$

$$\lim_{x\to 1^+}\left(\frac{\sqrt{x}-1}{x-1}\cdot\left(\frac{\sqrt{x}+1}{\sqrt{x}+1}\right)\right)$$

$$\lim_{x\to 1^+}\left(\frac{\cancel{x-1}}{\cancel{(x-1)}(\sqrt{x}+1)}\right) = \lim_{x\to 1^+}\left(\frac{1}{\sqrt{x}+1}\right)$$

$$\overset{DSP}{=} \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

$\lim_{x\to 1} f(x) = \frac{1}{2}$ because $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = \frac{1}{2}$