Trig Limits

Thursday, September 10, 2020 7:40 AM

EXAMPLE 10 **Trigonometric limits** Evaluate the following trigonometric limits.

a. $\lim_{x\to 0} \frac{\sin^2 x}{1 - \cos x}$ **b.** $\lim_{x\to 0} \frac{1 - \cos 2x}{\sin x}$

a) First try DSP

$$\lim_{x\to 0} \frac{\sin^2 x}{1-\cos x} \overset{\text{"DSP"}}{=} \frac{\sin^2 0}{\underbrace{1-\cos 0}_{1}} = \text{" } \frac{0}{0} \text{ "} \quad \text{indeterminate form}$$

Recall: $\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1^2 - \cos^2 x$

Recall: $a^2 - b^2 = (a-b)(a+b)$

$$\sin^2 x = (1-\cos x)(1+\cos x)$$

$$\lim_{x\to 0}\left(\frac{\sin^2 x}{1-\cos x}\right) = \lim_{x\to 0}\left(\frac{\cancel{(1-\cos x)}(1+\cos x)}{\cancel{1-\cos x}}\right) = \lim_{x\to 0}(1+\cos x)$$

$$\overset{\text{"DSP"}}{=} 1+\cos 0 = 1+1 = 2$$

EXAMPLE 10 **Trigonometric limits** Evaluate the following trigonometric limits.

a. $\lim_{x\to 0} \frac{\sin^2 x}{1 - \cos x}$ **b.** $\lim_{x\to 0} \frac{1 - \cos 2x}{\sin x}$

use the identity $\cos 2x = \cos^2 x - \sin^2 x$ to simplify the function.

$\sin^2 x + \sin^2 x$

$1 - \cos^2 x + \sin^2 x$

b) $$\lim_{x\to 0}\left(\frac{1-\cos 2x}{\sin x}\right) = \lim_{x\to 0}\left(\frac{1-(\cos^2 x - \sin^2 x)}{\sin x}\right)$$

$$\lim_{x\to 0}\left(\frac{2\cdot \sin^2 x}{\cancel{\sin x}}\right) = \lim_{x\to 0}\left(2\cdot \sin x\right) = 2\cdot \lim_{x\to 0}(\sin x)$$

$$\lim_{x\to 0}\left(\frac{2\cdot \sin x \cdot \cancel{\sin x}}{\cancel{\sin x}}\right)$$

$$\overset{\text{"DSP"}}{=} 2\cdot(\sin 0) = 0$$

21-Sep Midterm Exam #1: Ch. 1, App. B, 2.1, 2.2, 2.3

Given: Expand

45–50. Properties of logarithms *Assume* $\log_b x = 0.36$, $\log_b y = 0.56$, *and* $\log_b z = 0.83$. *Evaluate the following expressions.*

$\log_b\left(\frac{b^2 x^{5/2}}{\sqrt{y}}\right) = \log_b b^2 + \log_b x^{5/2} - \log_b \sqrt[2]{y}$

$= 2 \cdot \log_b b + \frac{5}{2} \cdot \log_b x - \log_b y^{1/2} = 2 + \frac{5}{2} \cdot (0.36) - \frac{1}{2} \cdot (0.56)$

$= 2.62$

$\log_a a = 1, \log_{100} 100 = 1$

Solve the equation:

Condense

44. $\ln(3x) + \ln(x+2) = 0$

$\ln_e(3x \cdot (x+2)) = 0$

$3x \cdot (x+2) = e^0 = 1 \Rightarrow 3x^2 + 6x = 1 \Rightarrow 3x^2 + 6x - 1 = 0$

Recall: $ax^2 + bx + c = 0$ $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 3$
$b = 6$
$c = -1$

$x_{1,2} = \frac{-6 \pm \sqrt{36 - 4 \cdot 3 \cdot (-1)}}{2 \cdot 3}$

$= \frac{-6 \pm \sqrt{36 + 12}}{6}$

$= \frac{-6 \pm \sqrt{48}}{6} = \frac{-6 \pm 4\sqrt{3}}{6}$

$x_1 = \frac{-6 + 4\sqrt{3}}{6}$

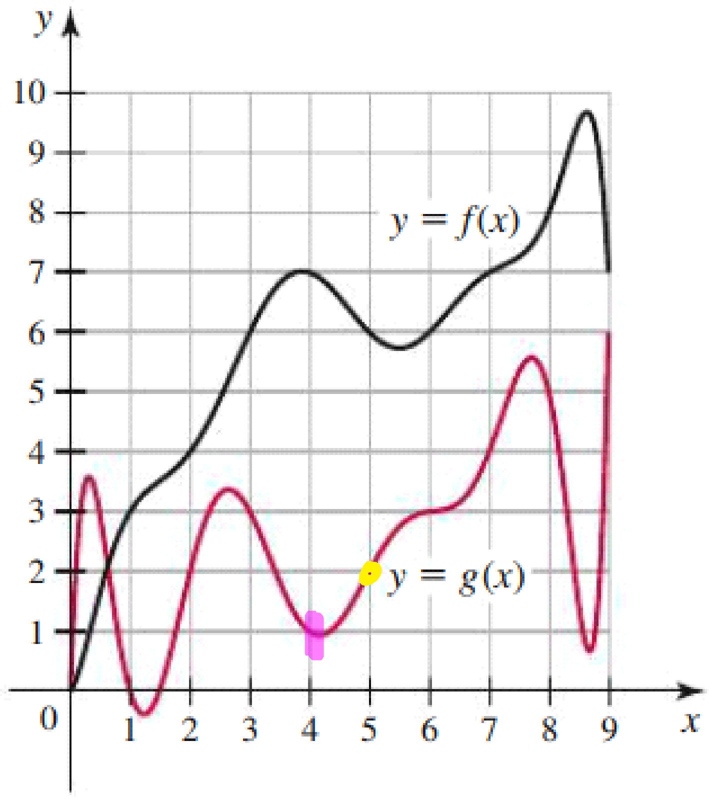
$x_2 = \frac{-6 - 4\sqrt{3}}{6}$

x_2 is neg. solution

$\ln(3x) \to \ln(\text{neg})$

15. Use the graphs of f and g in the figure to determine the following function values.

a. $(f \circ g)(2)$ **b.** $g(f(2))$
c. $f(g(4))$ **d.** $g(f(5))$
e. $f(f(8))$ **f.** $g(f(g(5)))$



a) $(f \circ g)(2) = f(\underbrace{g(2)}_{2})$

$= f(2) = 4$ ✓

e) $f(\underbrace{f(8)}) = f(8) = 8$ ✓

$f(8) = 8$

f) $g(f(\underbrace{g(5)})) = g(f(2)) = g(4) = 1$ ✓

27–30. Domain *State the domain of the function.*

27. $h(u) = \sqrt[3]{u - 1}$ **28.** $F(w) = \sqrt[4]{2 - w}$

29. $f(x) = (9 - x^2)^{3/2}$ **30.** $g(t) = \dfrac{1}{1 + t^2}$

$\sqrt[3]{0} = 0$

$\sqrt[5]{-32} = -2$

27) $\sqrt[3]{-8}$ vs. $\sqrt[2]{-8} = (-8)^{1/2}$

-2 vs. Not real #

Domain of $h(u)$ is ARN (All real numbers) $(-\infty, \infty)$

30) $g(t) = \dfrac{1}{1+t^2}$ $1 + t^2 \neq 0$
$t^2 \neq -1$

Domain of $g(t)$ is $(-\infty, \infty)$

97–100. Difference quotients *Simplify the difference quotients* $\frac{f(x+h)-f(x)}{h}$ *and* $\frac{f(x)-f(a)}{x-a}$ *by rationalizing the numerator.*

hint

99. $f(x) = -\frac{3}{\sqrt{x}}$

22. $f(x) = 4 - 5x$

99) $f(x+h) = \frac{-3}{\sqrt{x+h}}$

$f(x) = \frac{-3}{\sqrt{x}}$

$$\frac{\frac{2}{9}}{\frac{11}{1}} = \frac{2}{9 \cdot 11}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{-3}{\sqrt{x+h}}(\sqrt{x}) + \left(\frac{+3}{\sqrt{x}}\right)(\sqrt{x+h})}{h}$$

$$= \frac{\frac{-3\sqrt{x}}{\sqrt{x}\cdot(\sqrt{x+h})} + \frac{3(\sqrt{x+h})}{\sqrt{x}\cdot(\sqrt{x+h})}}{h} = \frac{\frac{-3(\sqrt{x}-\sqrt{x+h})}{\sqrt{x}\cdot(\sqrt{x+h})}}{\frac{h}{1}}$$

conjugate

$(a-b)(a+b) = a^2 - b^2$

$$= \frac{-3(\sqrt{x}-\sqrt{x+h})}{h\cdot\sqrt{x}(\sqrt{x+h})} \cdot \frac{(\sqrt{x}+\sqrt{x+h})}{(\sqrt{x}+\sqrt{x+h})}$$

$$= \frac{-3(x-(x+h))}{h\cdot\sqrt{x}(\sqrt{x+h})\cdot(\sqrt{x}+\sqrt{x+h})} = \frac{+3(\cancel{x}-\cancel{x}+\cancel{h})}{\cancel{h}\sqrt{x}(\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}$$

$$= \frac{3}{\sqrt{x}(\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}$$

Spring 2018 Midterm#1 Question

3. For each limit, calculate the value or show that it does not exist. Show all work.

(a) $\lim_{x \to 0} \left(\frac{(2x+9)^2 - 81}{x} \right)$ $\rightarrow$ $(2x+9)^2 - 9^2 = (2x+9+9)(2x+9-9)$
$= (2x+18) \cdot 2x$

(b) $\lim_{x \to 3^-} \left(\frac{|x-3|}{x-3} \right)$

(c) $\lim_{x \to 1} \left(\frac{5 - \sqrt{32 - 7x}}{x-1} \right)$

3a) let's try DSP $\frac{(2 \cdot 0 + 9)^2 - 81}{0} = \text{"}\frac{0}{0}\text{"}$ indeterminate form

$\lim_{x \to 0} \left(\frac{(2x+18) \cdot 2\cancel{x}}{\cancel{x}} \right) = \lim_{x \to 0} \left((2x+18) \cdot 2 \right)$

$\overset{\text{"DSP"}}{=} (2 \cdot 0 + 18) \cdot 2 = 36$

3b) $\lim_{x \to 3^-} \left(\frac{|x-3|}{x-3} \right)$

LL

let's say: 3

$|2.9 - 3| = -(-0.1)$

Since $x \to 3^-$ $|x-3| = -(x-3)$

$x \to 3^+$ $|x-3| = +(x-3)$

$\lim_{x \to 3^-} \left(\frac{-\cancel{(x-3)}}{\cancel{x-3}} \right) = -1$