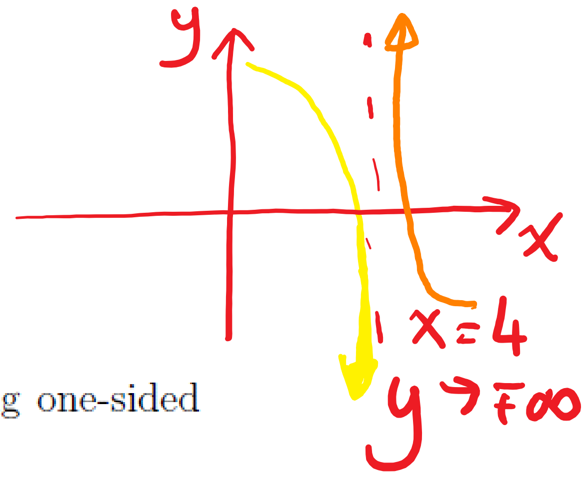
Midterm#2 Review: Ch 2.4, 2.5, 2.6

Monday, September 28, 2020 6:49 AM



① Find the vertical asymptote of $f(x) = \dfrac{x^2+16}{x^3-64}$. ② Then calculate the corresponding one-sided limits of f at the vertical asymptote.

Hint: $x^3-64 = x^3-4^3 = (x-4)(x^2+4x+16)$

Part1: $f(x) = \dfrac{x^2+16}{(x-4)(x^2+4x+16)}$ ←

V.A

$x-4=0 \Rightarrow \boxed{x=4}$

$x^2+4x+16=0$

$x_{1,2} = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ <0

$a=1, b=4, c=16$

$4^2-4\cdot1\cdot16 < 0$

No real zeros!

Part2: $\lim\limits_{x\to4^-} f(x)$, $\lim\limits_{x\to4^+} f(x)$

$4^- \to 4 \leftarrow 4^+$

DSP

$$\lim_{x\to4^-} \frac{\overbrace{x^2+16}^{\text{pos. \#}}}{\underbrace{(x-4)}_{\substack{\text{neg. \#}\\ \text{app. 0}}}\underbrace{(x^2+4x+16)}_{\text{pos. \#}}} = -\infty$$

DSP

$$\lim_{x\to4^+} \frac{\overbrace{x^2+16}^{\text{pos. \#}}}{\underbrace{(x-4)}_{\substack{\text{pos. \#}\\ \text{app. 0}\\ 0^+}}\underbrace{(x^2+4x+16)}_{\text{pos. \#}}} = \infty$$

DSP

$$\lim_{x\to4^-}\frac{\overbrace{x^2+16}^{\text{pos. \#}}}{\underbrace{(x-4)}_{\substack{\text{neg. \#}\\\text{app. 0}}}\underbrace{(x^2+4x+16)}_{\text{pos. \#}}}=-\infty$$

$$\lim_{x\to4^+}\frac{\overbrace{x^2+16}^{}}{\underbrace{\underbrace{(x-4)}_{\substack{\text{pos. \#}\\\text{app. 0}}}}_{0^+}\underbrace{(x^2+4x+16)}_{\text{pos. \#}}}=\infty$$

Evaluate

Exp) $\lim_{x\to\frac{\pi}{4}^+} \frac{\sec x}{\tan x-1}$

Recall:

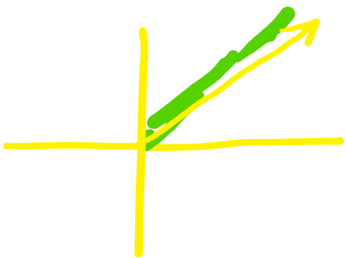
$\sec x = \frac{1}{\cos x}$

$\tan x = \frac{\sin x}{\cos x}$

$$\lim_{x\to\frac{\pi}{4}^+}\left(\frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}-\frac{1}{1}\,(\cos x)}\right)=\lim_{x\to\frac{\pi}{4}^+}\left(\frac{\cancel{\frac{1}{\cos x}}}{\frac{\boxed{\sin x-\cos x}}{\cancel{\cos x}}}\right)$$

$1=\frac{\cos x}{\cos x}$

$$\lim_{x\to\frac{\pi}{4}^+}\left(\frac{1}{\sin x-\cos x}\right)\overset{\text{DSP}\;''\quad''}{=}\frac{1}{\underset{\text{app. }0}{\text{pos. \#}}}=+\infty$$



$\boxed{x\to\frac{\pi}{4}^+}\;(\sin x-\cos x)\to 0^+$

$x=\frac{\pi}{3}$

$\sin\frac{\pi}{3}-\cos\frac{\pi}{3} > 0$ $\overset{0^-}{\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)}$ >0

QI: $\sin x\to 0\nearrow 1$ incr.

$\cos x\to 1\searrow 0$ decr.

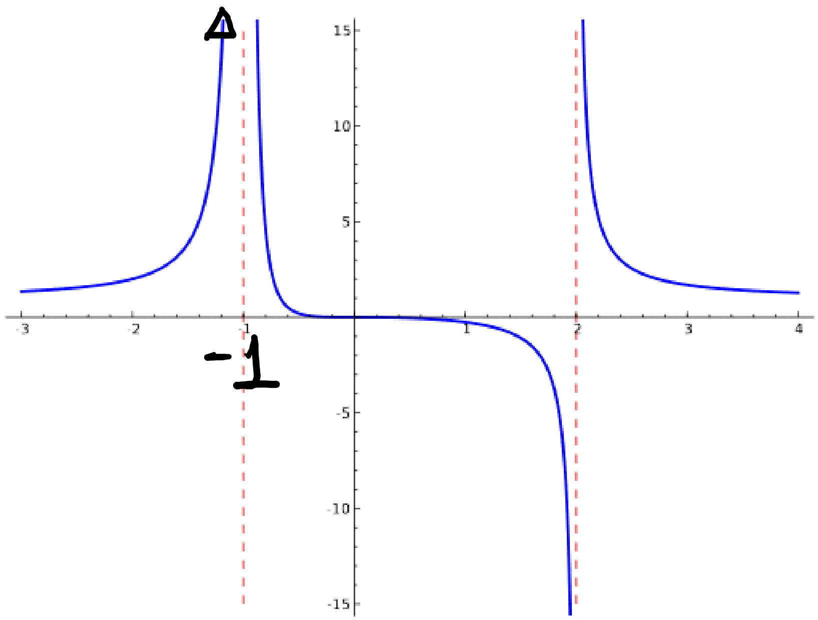
Recall:

$\sin\frac{\pi}{4}=\cos\frac{\pi}{4}$

Inf. Limits with Graph

Monday, September 28, 2020 7:43 AM

Consider the graph of $f(x)$ given below and compute the limits:



(a) $\lim_{x\to -1^-} f(x) = \infty$

(b) $\lim_{x\to -1^+} f(x) = \infty$

$\lim_{x\to -1} f(x) = \infty$

(c) $\lim_{x\to 2^-} f(x) = -\infty$

(d) $\lim_{x\to 2^+} f(x) = \infty$

$\lim_{x\to 2} f(x)$ DNE

Problem 2. (3 points) For what value of $\underline{\underline{a}}$ is the following function continuous for all x? If this is not possible, explain why.

$$f(x) = \begin{cases} -5ax & x \neq 3 \rightarrow \text{piece\#1} \\ \sin(\frac{\pi x}{2}) & x = 3 \rightarrow \text{piece\#2} \end{cases}$$

Transition P. at $\underline{x=3}$

3 checklist (Is f(x) cont. at x=b?)

1) $f(b) \rightarrow f(3) = \sin\left(\frac{\pi \cdot 3}{2}\right) = \sin\left(\frac{3\pi}{2}\right) = -1$

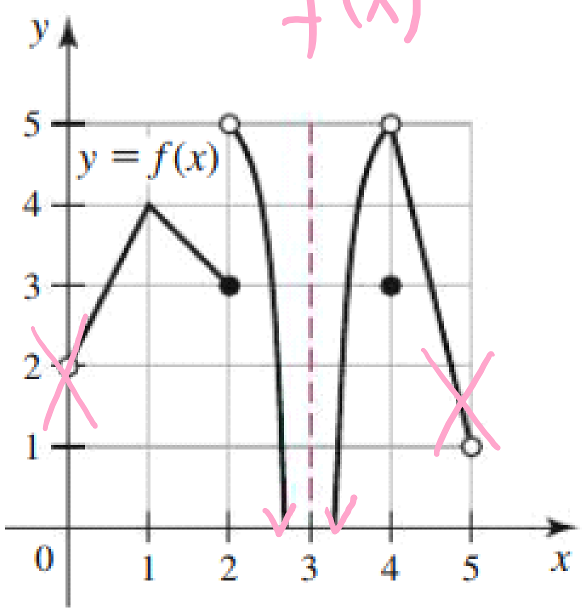
2) $\lim_{x\to b} f(x) \rightarrow \lim_{x\to 3\,(x\neq 3)} f(x) = \lim_{x\to 3}(-5ax) \overset{\text{"DSP"}}{=} -15a$

3) $\left.f(b) = \lim_{x\to b} f(x)\right\}$ $\quad f(3) = \lim_{x\to 3} f(x)$

$$\frac{-1}{-15} = \frac{-15a}{-15} \Rightarrow a = \frac{1}{15}$$

Determine the points of discontinuity on (0,5)

f(x)



@ x=2 $\lim_{x\to 2} f(x)$ DNE

@ x=3 f(3) is undef.

@ x=4 $\left.\begin{matrix}\lim_{x\to 4} f(x) = 5 \\ f(4) = 3\end{matrix}\right\}$ $5 \neq 3$

Continuity

Monday, September 28, 2020 7:00 AM

Exp5)

$$f(x)=\begin{cases} \left(\frac{\sin(Ax)}{x}\xrightarrow{} x\neq 0\right), & \text{if } \boxed{x<0} \\ 5, & \text{if } x=0 \\ x^3+B, & \text{if } \boxed{x>0} \end{cases}
\qquad
\begin{array}{c} \underline{\text{Hint}} \\ 0^- \\ \\ 0^+ \end{array}
\qquad
\begin{array}{c} \checkmark \\ \checkmark \\ \checkmark \end{array}$$

Find the values of A and B which make f(x) continuous.

check the transition point, $x=0$

1) $f(0)=5$

2) $\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^-}\frac{\sin(Ax)}{x}$; $\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0^+}(x^3+B)$

$$\left.\lim_{x\to 0^-}\frac{\sin(Ax)}{x}=\lim_{x\to 0^+}(x^3+B)\right\}\ \lim_{x\to 0} f(x) \text{ exist}$$

$$\left.\begin{array}{l} \lim\limits_{x\to 0^-} \boxed{\frac{A\cdot\sin(Ax)}{A\cdot x}} \xrightarrow{} 1 \quad = A \\ \\ \lim\limits_{x\to 0^+}(x^3+B) \overset{\text{"DSP"}}{=} 0^3+B=B \end{array}\right\}\quad A=B$$

3) $$\left. f(0)=\lim_{x\to 0} f(x)\right\}\ \boxed{5=A=B}$$

Limits at Infinity

Monday, September 28, 2020 7:02 AM

Calculate $\lim_{x\to-\infty}\left(\frac{\sqrt{3x^2-6}}{5+2x}\right)$

Take out x^2 from $(3x^2-6)$ first

$$\lim_{x\to-\infty}\left(\frac{\sqrt{x^2\left(3-\frac{6}{x^2}\right)}}{5+2x}\right)=\lim_{x\to-\infty}\left(\frac{|x|\cdot\sqrt{3-\frac{6}{x^2}}}{5+2x}\right)$$

Take out x from $(5+2x)$ (recall: $x\to-\infty$ $|x|=-x$)

$$\lim_{x\to-\infty}\left(\frac{-\cancel{x}\cdot\sqrt{3-\frac{6}{x^2}}}{\cancel{x}\left(\frac{5}{x}+2\right)}\right)=\lim_{x\to-\infty}\left(\frac{-\sqrt{3-\cancel{\frac{6}{x^2}}}}{\cancel{\frac{5}{x}}+2}\right)$$

$$=\lim_{x\to-\infty}\left(\frac{-\sqrt{3}}{2}\right)\boxed{=\frac{-\sqrt{3}}{2}}$$

Limits at Inf (e^x)

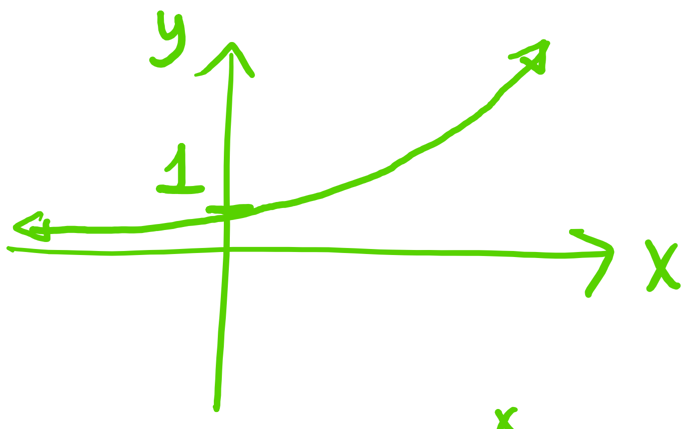
Monday, September 28, 2020 7:04 AM

Calculate $$\lim_{x\to\infty}\left(\frac{e^x-1}{4+5e^x}\right)$$

Take out e^x from the numerator & denominator:

$$\lim_{x\to\infty}\left(\frac{\cancel{e^x}\left(1-\cancelto{0}{\frac{1}{e^x}}\right)}{\cancel{e^x}\left(\cancelto{0}{\frac{4}{e^x}}+5\right)}\right)$$

Recall: $f(x)=e^x$



$x\to\infty$, $e^x\to\infty$

$$\lim_{x\to\infty}\left(\frac{1}{5}\right)=\frac{1}{5}$$

Dr. Tabanli's Spring 2020 Exam2 Question:

Thursday, September 17, 2020 8:11 AM

Find all horizontal asymptotes of $f(x) = \dfrac{8-3x}{2x+\sqrt{25x^2+x+13}}$. Write "NONE" if f has no horizontal asymptotes.

Step1) Identify the highest (degree) exponent in the denom.

Step2) Divide ALL by

USE LIMITS!

Step1) x^1 is the highest exponent in the denom.

Case 1

$$\lim_{x\to\infty}\left(\frac{\frac{8-3x}{x}}{\frac{2x+}{x}\sqrt{\frac{25x^2+x+13}{x^2}}}\right) = \lim_{x\to\infty}\left(\frac{\overset{0}{\cancel{\frac{8}{x}}}-3}{2+\sqrt{25+\overset{0}{\cancel{\frac{1}{x}}}+\overset{0}{\cancel{\frac{13}{x^2}}}}}\right)$$

$$\lim_{x\to\infty}\left(\frac{-3}{2+\sqrt{25}}\right) = \lim_{x\to\infty}\left(\frac{-3}{7}\right) = \boxed{\frac{-3}{7}}$$

$$\sqrt{x^2} = |x| \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Case 2

$$\lim_{x\to-\infty}\left(\frac{\frac{8-3x}{-x}}{\frac{2x}{-x}+\sqrt{\frac{25x^2+x+13}{x^2}}}\right) = \lim_{x\to-\infty}\left(\frac{\overset{0}{\cancel{\frac{-8}{x}}}+3}{-2+\overset{5}{\sqrt{25+\cancel{\frac{1}{x}}+\cancel{\frac{13}{x^2}}}}}\right)$$

$$= \lim_{x\to-\infty}\left(\frac{3}{-2+5}\right) = \boxed{1}$$

Alternative (more Practical) method:

Find all horizontal asymptotes of $f(x) = \dfrac{8-3x}{2x+\sqrt{25x^2+x+13}}$. Write "NONE" if f has no horizontal asymptotes.

Focused on factoring more:

$$f(x) = \frac{8-3x}{2x+\sqrt{x^2\left(25+\frac{1}{x}+\frac{13}{x^2}\right)}} = \frac{8-3x}{2x+\underbrace{|x|}\cdot\sqrt{25+\frac{1}{x}+\frac{13}{x^2}}}$$

recall $\sqrt{x^2}=|x|$

Case#1 As $x \to \infty$ $|x| = x$;

$$\lim_{x\to\infty} \frac{8-3x}{2x+x\sqrt{25+\frac{1}{x}+\frac{13}{x^2}}} = \lim_{x\to\infty} \frac{\cancel{x}\left(\overset{0}{\cancel{\frac{8}{x}}}-3\right)}{\cancel{x}\left(2+1\cdot\sqrt{25+\overset{0}{\cancel{\frac{1}{x}}}+\overset{0}{\cancel{\frac{13}{x^2}}}}\right)}$$

$$= \lim_{x\to\infty}\left(\frac{-3}{2+5}\right) = \boxed{\frac{-3}{7}}$$

Case#2 As $x \to -\infty$ $|x| = -x$;

$$\lim_{x\to-\infty} \frac{8-3x}{2x-x\sqrt{25+\frac{1}{x}+\frac{13}{x^2}}} = \lim_{x\to-\infty} \frac{\cancel{x}\left(\overset{0}{\cancel{\frac{8}{x}}}-3\right)}{\cancel{x}\left(2-1\cdot\sqrt{25+\overset{0}{\cancel{\frac{1}{x}}}+\overset{0}{\cancel{\frac{13}{x^2}}}}\right)}$$

$$= \lim_{x\to-\infty}\left(\frac{-3}{2-5}\right) = \frac{-3}{-3} = \boxed{1}$$