Midterm#2 Review - Part 2

Monday, September 28, 2020 6:59 AM

Find the values of A and B which make $f(x)$ continuous for all x; or explain why such values do not exist.

$$f(x)=\begin{cases} Ax+3, & \text{if } x<1 \quad \text{linear} \\ 5, & \text{if } x=1 \quad \text{constant} \\ x^2+B, & \text{if } x>1 \quad \text{quadratic} \end{cases}$$

continuous

check 3 conditions of continuity at $x=1$

1) $f(1) = 5$

2) $\lim\limits_{x\to 1} f(x)$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} (Ax+3) \overset{\text{"DSP"}}{=} A+3$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} (x^2+B) = 1^2+B = 1+B$

$A+3 = 1+B$ for $\lim\limits_{x\to 1} f(x)$ to exist.

3) $f(1) = \lim\limits_{x\to 1} f(x)$ } $5 = A+3 = 1+B$

$A=2,\ B=4$

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Evaluate $\lim_{x \to -\infty} \left(\frac{2x - \sqrt{9x^2 - 4}}{8x+3} \right)$

more practical method: deal w/ the radical first!

$$\lim_{x \to -\infty} \left(\frac{2x - \sqrt{x^2\left(9 - \frac{4}{x^2}\right)}}{8x+3} \right)$$

recall:
$x \to -\infty$
$(x < 0)$
$\sqrt{x^2} = |x| = -x$

$$\lim_{x \to -\infty} \left(\frac{2x + (+x)\sqrt{9 - \frac{4}{x^2}}}{8x+3} \right)$$

$$\lim_{x \to -\infty} \left(\frac{2x + x\sqrt{9 - \frac{4}{x^2}}}{8x+3} \right)$$

Divide ALL by x

$$\lim_{x \to -\infty} \left(\frac{\frac{2x}{x} + \frac{x\sqrt{9 - \frac{4}{x^2}}}{x}}{\frac{8x}{x} + \frac{3}{x}} \right) = \lim_{x \to -\infty} \left(\frac{2 + \sqrt{9 - \cancel{\frac{4}{x^2}}^{\,0}}}{8 + \cancel{\frac{3}{x}}^{\,0}} \right)$$

$$\lim_{x \to -\infty} \left(\frac{2 + \sqrt{9}}{8 + 0} \right) = \frac{5}{8}$$

recall: $\lim_{x \to \pm\infty} \left(\frac{1}{x^n} \right) = 0$

Limits at Inf (e^x)

Monday, September 28, 2020 7:04 AM

Calculate $\lim_{x\to\infty}\left(\frac{e^x-1}{4+5e^x}\right)$

Divide both numerator and denominator by e^x:

$$\lim_{x\to\infty}\left(\frac{\frac{e^x-1}{e^x}}{\frac{4+5e^x}{e^x}}\right)=\lim_{x\to\infty}\left(\frac{1-\frac{1}{e^x}}{\frac{4}{e^x}+5}\right)=\boxed{\frac{1}{5}}$$

Recall:

as $x\to\infty$ $\frac{1}{e^x}\to 0$

Dr. Tabanli's Past Exam Question#1

Wednesday, September 30, 2020 5:22 PM

Find the value of k that makes f continuous at $x = 1$ or determine that no such value of k exists. If there is no k value exists, write "DNE".

$$f(x) = \begin{cases} kx^3 + e^{x-1} & , \quad x < 1 \\ 3x - \ln(2x-1) & , \quad x \geq 1 \end{cases}$$

Conditions for continuity of $f(x)$ at a point $x=1$

1) $f(1) = 3 \cdot 1 - \ln(2 \cdot 1 - 1) = 3 - \cancel{\ln 1}^{0} = 3$

2) $\lim_{x \to 1} f(x) \to \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (kx^3 + e^{x-1}) \stackrel{DSP}{=} k \cdot 1^3 + e^{1-1} = k + \cancel{e^0}^{1} = k+1$

$\lim_{x \to 1} f(x) \Leftarrow LL = RL \quad (k+1 = 3 \Rightarrow k = 2)$

$\to \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (3x - \ln(2x-1)) \stackrel{DSP}{=} 3 - \cancel{\ln(1)}^{0} = 3$

3) 1) = 2) $\quad$ $f(1) = 3$, $\lim_{x \to 1} f(x) = 3$

$3 = 3$ ✓

All 3 conditions must be true if $f(x)$ is continuous at $x = 1$.

$k = 2$

Dr. Tabanli's Past Exam Question#2

Wednesday, September 30, 2020 5:22 PM

$f(1)$ is irrelevant

Evaluate the limit or determine that it does not exist. If the limit does not exist, write "DNE".
You must use proper calculus methods and notation to receive full credit.

$$\lim_{x\to 1} f(x)\ ,\ \text{where}\quad f(x) = \begin{cases} \dfrac{\sqrt{x}-1}{x-1} & ,\ x > 1 \\ 8 & ,\ x = 1 \\ \dfrac{2x-2}{x^2+2x-3} & ,\ x < 1 \end{cases}$$

RL

LL

check

$\lim_{x\to 1^-} f(x)$, $\lim_{x\to 1^+} f(x)$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{(2x-2)}{(x^2+2x-3)} = \lim_{x\to 1^-} \frac{2\cancel{(x-1)}}{(x+3)\cancel{(x-1)}}$$

$$= \lim_{x\to 1^-} \frac{2}{x+3} \overset{DSP}{=} \frac{2}{1+3} = \frac{2}{4} = \frac{1}{2}$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{\sqrt{x}-1}{x-1} = \lim_{x\to 1^+} \frac{\cancel{\sqrt{x}-1}}{\cancel{(\sqrt{x}-1)}(\sqrt{x}+1)}$$

$$(x-1 = (\sqrt{x})^2 - 1^2 = (\sqrt{x}-1)(\sqrt{x}+1))$$

$$= \lim_{x\to 1^+} \frac{1}{\sqrt{x}+1} \overset{DSP}{=} \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

Since: $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = \frac{1}{2}$ $\Big\}$ $\boxed{\lim_{x\to 1} f(x) = \frac{1}{2}}$

Dr. Tabanli's Past Exam Question#3

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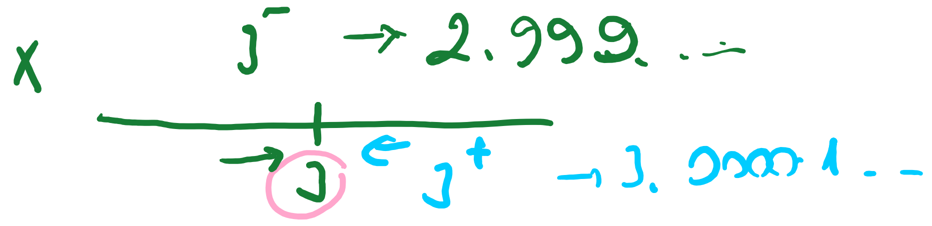
For each part, calculate the limit or show that it does not exist. If the limit is infinite, write "∞" or "$-\infty$" as your answer, as appropriate.

(a) $\lim_{x\to 3^-}\left(\dfrac{x^2+6}{3-x}\right)$

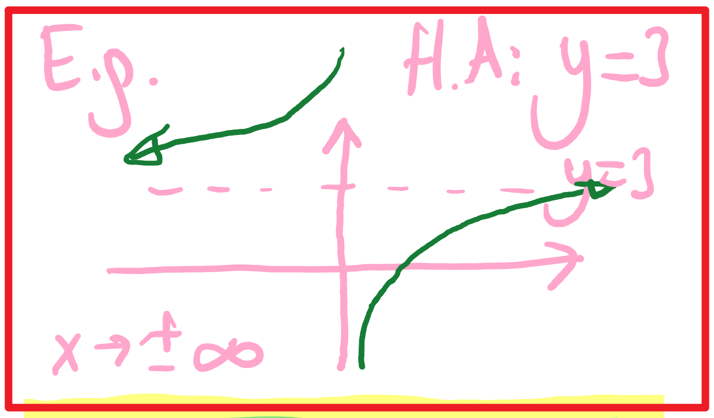


DSP $3^2+6=15$ pos. non-zero #

$$\lim_{x\to 3^-}\left(\frac{x^2+6}{3-x}\right)=\infty$$

pos. # app. 0

Recall:
End behavior

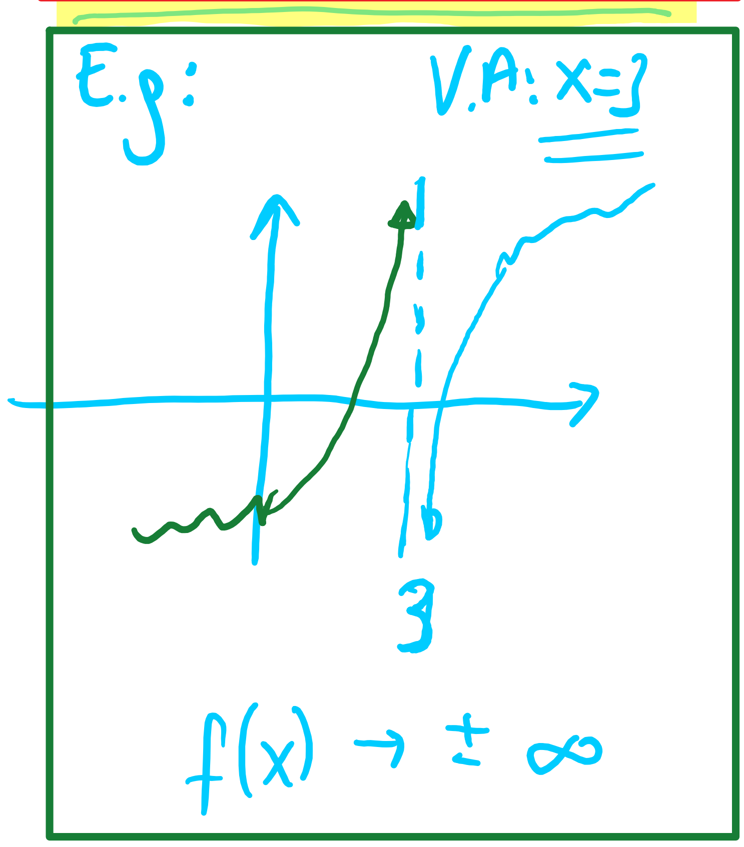


Evaluate $\lim_{x\to 3^+}\left(\dfrac{x^2+6}{3-x}\right)$

DSP (pos. #)

$$\lim_{x\to 3^+}\left(\frac{x^2+6}{3-x}\right)=-\infty$$

neg. # app. 0



Evaluate $\lim_{x\to 3}\left(\dfrac{x^2+6}{3-x}\right)$

Since $\lim_{x\to 3^+}\left(\dfrac{x^2+6}{3-x}\right)\neq\lim_{x\to 3^-}\left(\dfrac{x^2+6}{3-x}\right)$

$\lim_{x\to 3}\left(\dfrac{x^2+6}{3-x}\right)$ DNE

Let $f(x) = \dfrac{x^2 + x - 6}{x^2 - x - 12}$.

a. **Determine the point(s) of discontinuity. Write the interval of continuity in interval notation.**

b. **Determine the vertical asymptotes of f(x) if any.**

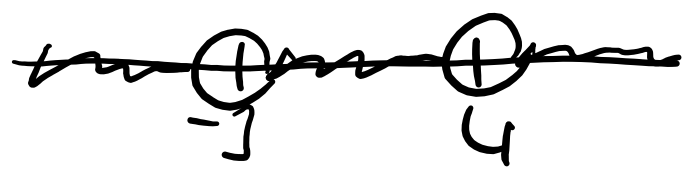
c. **Determine the horizontal asymptotes of f(x) if any.**

a. Factor: $f(x) = \dfrac{(x+3)(x-2)}{\underbrace{(x-4)}_{0}\underbrace{(x+3)}_{0}}$

f is rational, therefore, it's continuous for all x at which the denominator is nonzero.

f is continuous for all x except $x=4$, $x=-3$.

Interval of continuity:



$(-\infty, -3), (-3, 4), (4, \infty)$

b. Determine V.A after simplifying f(x)

$$f(x) = \frac{\cancel{(x+3)}(x-2)}{(x-4)\cancel{(x+3)}} = \frac{x-2}{x-4}$$

zero of the simplified denominator is $x=4$;

verify!

$$\lim_{x \to 4^-} \left(\frac{\overbrace{x-2}^{\text{app. 2 (pos. \#)}}}{\underbrace{x-4}_{\text{neg. \# app. 0}}} \right) = -\infty$$

$$\lim_{x \to 4^+} \left(\frac{\overbrace{x-2}^{\text{app. 2 (pos. \#)}}}{\underbrace{x-4}_{\text{pos. \# app. 0}}} \right) = \infty$$

Function has a V.A. at $x=4$.

C. Divide ALL by x^2, as $x \to \pm\infty$

Let $f(x) = \dfrac{x^2 + x - 6}{x^2 - x - 12}$.

$$\lim_{x\to\infty}\left(\frac{\frac{x^2+x-6}{x^2}}{\frac{x^2-x-12}{x^2}}\right) = \lim_{x\to\infty}\left(\frac{1+\cancel{\frac{1}{x}}^{\nearrow 0}-\cancel{\frac{6}{x^2}}^{\nearrow 0}}{1-\cancel{\frac{1}{x}}^{\nearrow 0}-\cancel{\frac{12}{x^2}}^{\nearrow 0}}\right)$$

$$\lim_{x\to\infty}\left(\frac{1}{1}\right) = 1$$

recall:

$$\lim_{x\to\pm\infty}\left(\frac{1}{x^n}\right) = 0$$

$$\lim_{x\to-\infty}\left(\frac{1+\cancel{\frac{1}{x}}^{\nearrow 0}-\cancel{\frac{6}{x^2}}^{\nearrow 0}}{1-\cancel{\frac{1}{x}}^{\nearrow 0}-\cancel{\frac{12}{x^2}}^{\nearrow 0}}\right) = 1.$$

The graph has the H.A. $y = 1$.